Oscillator design using two-port describing functions

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***Abstract*— Our goal is to show that the describing function concept is useful for oscillator design. Describing functions have been extended for the case of two-ports. Some properties of two-port describing functions have been presented. Two-port admittance describing functions of a simplified transistor model have been determined. As an application, a Colpitts oscillator has been designed for the MHz range. There is a good coincidence between circuit analysis and measurements, and these results deviate from describing function method. This indicates a modelling problem that will be solved in our next publication.**

***Index Terms*— Oscillator, nonlinearity, describing function concept, comparison, AWR, measurement**

I. Introduction: Two-port describing functions, properties

Describing function concept dates back as long as to 1968 when the famous monograph by Gelb and van der Velde appeared [1]. A describing function is an approximate characterization of a nonlinear operator when sinusoidal answer for sinusoidal excitation as a function of signal intensity is considered. The cited reference generalizes the concept for multiple inputs. In our early work, so called two-port admittance and scattering describing functions have been introduced [2]. Input and output currents are described as functions of the magnitude of input and output voltages and the phase between them. Another generalization is the describing function matrix, that is, generalization of the describing function concept for multiple harmonics [3]. When both multiple ports and multiple harmonics are considered, the characterization is called as X-parameters [4] assuming that the nonlinearity is concentrated on the first port only.

In this paper, application of the two-port describing functions for the case of oscillator design has been studied. Two-port admittance describing functions for a simplified transistor model have been derived in Section II. Then these results have been used in the design of a low frequency Colpitts oscillator in Section III.

Now we define two-port describing functions and summarize their properties.

A nonlinear time-invariant, dynamic two-port [5] can be characterized approximately by sinusoidal input two-port describing functions in terms of the admissible signal pairs ξ_i, η_i (i=1, 2) for the ith port. Admissible signal pairs [5] can be voltage and current, incident and reflected wave parameters, or any linear combinations of the voltage and current time functions. Later on, we use admittance describing functions where admissible signal pair is current and voltage. In our paper [2], we show a microwave measurement example where admissible signal pairs are reflected and incident wave parameters.

We assume that time dependence of the admissible signal pairs is sinusoidal. This is a consequence of some tuned circuits included in the embedding circuit around the two-port. From this point, we denote by ξ and η, the complex effective values of the port quantities.

Then two-port describing functions are defined as follows:

$$\xi_1 = DF_1(|\eta_1|, |\eta_2|, \varphi)\eta_1 \tag{1}$$

$$\xi_2 = DF_2(|\eta_1|, |\eta_2|, \varphi)\eta_1 \tag{2}$$

Note that two-port describing functions can be defined in many different ways. This definition expresses the fact that the effect of our two-port to the environment is a reflection (DF_1) and a transmission (DF_2).

In the Eqs. (1), (2), φ is the phase difference between η_2 and η_1.

As shown in Eqs. (1) and (2), we consider „input” and „transfer” as nonlinear functions of $|\eta_1|, |\eta_2|, \varphi$.

Next we consider the relation between describing functions and circuit parameters if the two-port is linear.

A linear two-port can be characterized as follows:

$$\xi_1 = CP_{11}\eta_1 + CP_{12}\eta_2 \tag{3}$$
$$\xi_2 = CP_{21}\eta_1 + CP_{22}\eta_2 \tag{4}$$

where CP_{ij} is the circuit parameter with excitation and answer at port j and I, respectively.

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The comparison between Eqs. (1,2) and (3,4) yields

$$DF_1 = CP_{11} + CP_{12}\eta_2/\eta_1 \quad (5)$$

$$DF_2 = CP_{21} + CP_{22}\eta_2/\eta_1 \quad (6)$$

Eq. (5) says that if the frequency is fixed, DF_1 is a circle with centre and radius CP_{11} and CP_{12}, respectively, if the amplitudes of η_2 and η_1 are identical and the phase between them is varied. Situation is very similar for DF_2.

Reciprocity implies $CP_{12} = CP_{21}$ so in case of the two circles, the centre of DF_2 is related to the radius of DF_1.

Symmetry implies reciprocity plus $CP_{11} = CP_{22}$ so the centre of DF_1 is related to the radius of DF_2.

Losslessness is described differently for different choice of admissible signal pairs, so losslessness is seen differently in terms of different describing functions.

In the following, we use admittance describing functions for low frequency design, and scattering describing functions for microwave design. For admittance describing functions, the admissible signal pair is current, voltage and for scattering describing functions, reflected and incident wave variables.

In today's design, mostly nonlinear two-ports (transistors) are used as an active element. Therefore, we define a possible generalization of the describing function concept for the case of low frequency two-ports [2]:

$$I_1 = Y_I(|V_1|, |V_2|, \varphi_V)V_1 \quad (7)$$

$$I_2 = Y_T(|V_1|, |V_2|, \varphi_V)V_1 \quad (8)$$

where I_1, V_1, I_2 and V_2 are the complex amplitudes of the first harmonic currents and voltages at the first and second ports with the reference directions given in Fig. 1, φ_V is the phase between the two voltage excitations. The indices I and T stand for input and transfer, respectively.

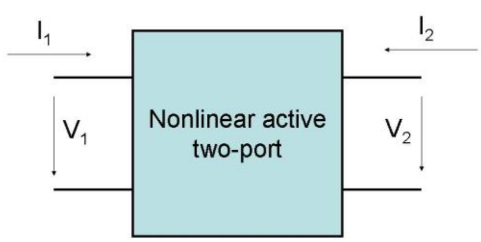

Figure 1. Reference directions

That means the nonlinear active two-port is characterized by two describing functions, one for the input and the other for the transfer from the first port to the second. As one phase can be arbitrarily chosen, a possibility is to choose V_1 as real and then φ_V is the phase of the complex voltage amplitude at the second port.

With these preliminary information, we determine the two-port describing functions of a nonlinear transistor model in Section III, and in Section IV we use them in the design of an oscillator.

Advantage of the describing function approach is that we can approximate well the nonlinear behavior even if it is strong. We expect that using this approach will result in better coincidence between our measured and analyzed parameters than before.

We use the following index conventions:

- Lowercase letter, uppercase index: General time function
- Uppercase letter, uppercase index: DC component
- Lowercase letter, lowercase index: Alternating component
- Uppercase letter, lowercase index: Complex effective value of the first harmonic

Next we derive the relations between admittance and scattering describing functions. Let us start with the definitions.

$$a = \frac{v+Z_0 i}{2\sqrt{Z_0}} \quad (9)$$

$$b = \frac{v-Z_0 i}{2\sqrt{Z_0}} \quad (10)$$

$$v = \sqrt{Z_0}(\mathrm{a+b}) \quad (11)$$

$$i = \frac{a-b}{\sqrt{Z_0}} \quad (12)$$

Substitution of Eqs. (11,12) into (7,8) and rearranging, results in the transformation from admittance to scattering description.

$$S_R = \frac{1-Y_I Z_0}{1+Y_I Z_0} \quad (13)$$

$$S_T = \frac{1-Y_T Z_0}{1+Y_T Z_0} \quad (14)$$

And the inverse transformation is:

$$Y_I Z_0 = \frac{1-S_R}{1+S_R} \quad (15)$$

$$Y_T Z_0 = \frac{1-S_T}{1+S_T} \quad (16)$$

Next we derive transformation between Q_V and Q_W:

$$Q_V = \frac{V_2}{V_1} \quad (17)$$

$$Q_W = \frac{A_2}{A_1} \quad [2] \quad (18)$$

Substituting Eq. (11) into (17):

$$Q_V = \frac{A_2+B_2}{A_1+B_1} = \frac{\frac{A_2}{A_1}+\frac{B_2}{A_1}}{1+\frac{B_1}{A_1}} = \frac{Q_W+S_T}{1+S_R} \quad (19)$$

II. TWO-PORT ADMITTANCE DESCRIBING FUNCTIONS OF A SIMPLIFIED TRANSISTOR MODEL

As the forthcoming calculation may be lengthy for a real transistor model, we have to propose the simplest possible model. First we intend to try a low frequency realization of the oscillator thus transistor parasitic elements are neglected all. Further simplification is possible if we use such circuit element values in our design that suppress effectively the influence of a real transistor input.

In the planned Colpitts oscillator, the transistor operates with grounded collector, thus the input and output are the basis and the emitter as shown in the following Figure.

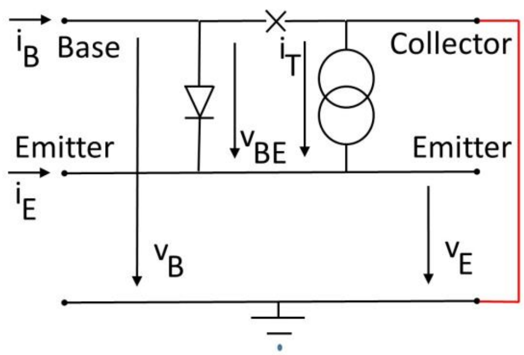

Figure 2. Transistor model (AC)

We assume a diode and a nonlinear voltage controlled current source with the following characteristic:

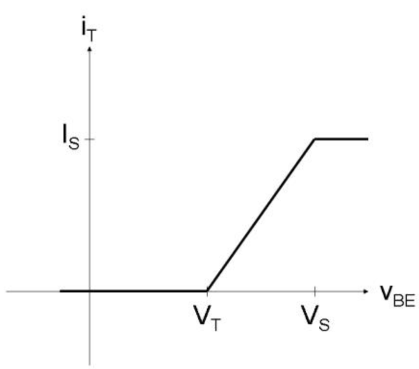

Figure 3. Nonlinear transconductance

$$i_T = \begin{cases} 0 & if \quad v_{BE} < V_T \\ I_S \dfrac{v_{BE} - V_T}{V_S - V_T} & if \quad V_T \le v_{BE} < V_S \\ I_S & if \quad V_S \le v_{BE} \end{cases} \quad (26)$$

It is easy to show that a circuit consisting 2 two-ports in parallel has the following two-port admittance describing functions:

$$Y_I(|V_b|,|V_e|,\varphi) = Y_{I1}(|V_b|,|V_e|,\varphi) + Y_{I2}(|V_b|,|V_e|,\varphi) \quad (27)$$

$$Y_T(|V_b|,|V_e|,\varphi) = Y_{T1}(|V_b|,|V_e|,\varphi) + Y_{T2}(|V_b|,|V_e|,\varphi) \quad (28)$$

Let the first two-port comprise the diodes only and the second comprise the nonlinear controlled source. Circuit equations of the first 2-port are

$$I_{b1} = I_E / B * I_1(V_{be} / nV_{TBE}) \quad (29)$$

$$I_{e1} = -I_E / B * I_1(V_{be} / nV_{TBE}) \quad (30)$$

where I_1 is the modified Bessel function of the first order, I_E and B are the DC emitter current and the DC current gain, respectively. Therefore the describing functions of the first two-port are:

$$Y_{I1} = \frac{I_{b1}}{V_b} \quad (31)$$

$$Y_{T1} = \frac{I_{e1}}{V_b} \quad (32)$$

Now we calculate the describing functions of the second two-port. It is obvious that Y_{I2}=0. Base-emitter voltage is

$$v_{BE} = V_{be}\cos(\omega_0 t) \quad (33)$$

where the amplitude is chosen as real. We have to calculate the amplitude of the first harmonic current at the second port. As the nonlinearity is resistive, the output current is in phase with the input voltage. For cosine excitation, there will be only cosine component of the first harmonic. There are three cases:

a. $V_{be} < V_T$ then

$$I_{e2} = 0 \quad (34)$$

b. $V_T \le V_{be} < V_S$ then

$$I_{e2} = -\frac{1}{2\pi}\int_{-\varphi_1}^{\varphi_1} I_S \frac{V_{be}\cos(\omega_0 t)-V_T}{V_S - V_T}\cos(\omega_0 t)d\omega_0 t \tag{35}$$

$$\varphi_1 = a\cos\left(\frac{V_T}{V_{be}}\right) \tag{36}$$

c. $V_S \le V_{be}$ then

$$\begin{aligned} \mathrm{I_{e2}} &= -\frac{1}{2\pi}\int_{-\varphi_3}^{-\varphi_2} \mathrm{I_S}\frac{\mathrm{V_{be}}\cos(\omega_0 \mathrm{t})-\mathrm{V_T}}{\mathrm{V_S}-\mathrm{V_T}}\cos(\omega_0 \mathrm{t})d\omega_0 \mathrm{t} - \\ &-\frac{1}{2\pi}\int_{-\varphi_2}^{\varphi_2}\mathrm{I_S}\cos(\omega_0 \mathrm{t})d\omega_0 \mathrm{t} - \\ &-\frac{1}{2\pi}\int_{\varphi_2}^{\varphi_3} \mathrm{I_S}\frac{\mathrm{V_{be}}\cos(\omega_0 \mathrm{t})-\mathrm{V_T}}{\mathrm{V_S}-\mathrm{V_T}}\cos(\omega_0 \mathrm{t})d\omega_0 \mathrm{t} \end{aligned} \tag{37}$$

$$\varphi_2 = a\cos\left(\frac{V_S}{V_{be}}\right) \tag{38}$$

$$\varphi_3 = a\cos\left(\frac{V_T}{V_{be}}\right) \tag{39}$$

Now we evaluate the b. and c. cases.

b.

$$\begin{aligned} &\int_{-\varphi_1}^{\varphi_1} I_S \frac{V_{be}\cos(\omega_0 t)-V_T}{V_S - V_T}\cos(\omega_0 t)d\omega_0 t = \\ &= \frac{I_S}{V_S - V_T}\left[\frac{V_{be}}{2}\omega_0 t + \frac{V_{be}}{4}\sin 2\omega_0 t - V_T \sin\omega_0 t\right]_{-\varphi_1}^{\varphi_1} = \\ &= \frac{I_S}{V_S - V_T}\left[V_{be}\varphi_1 + \frac{V_{be}}{2}\sin 2\varphi_1 - 2V_T\sin\varphi_1\right] \\ &= \frac{I_S}{V_S - V_T}\left[V_{be}a\cos\left(\frac{V_T}{V_{be}}\right) + \frac{V_{be}}{2}\sin\left(2a\cos\left(\frac{V_T}{V_{be}}\right)\right) - 2V_T\sin a\cos\left(\frac{V_T}{V_{be}}\right)\right] = \\ &= \frac{I_S}{V_S - V_T}\left[V_{be}a\cos\left(\frac{V_T}{V_{be}}\right) + V_{be}\frac{V_T}{V_{be}}\sqrt{1-\left(\frac{V_T}{V_{be}}\right)^2} - 2V_T\sqrt{1-\left(\frac{V_T}{V_{be}}\right)^2}\right] = \\ &= \frac{I_S}{V_S - V_T}\left[V_{be}a\cos\left(\frac{V_T}{V_{be}}\right) - V_T\sqrt{1-\left(\frac{V_T}{V_{be}}\right)^2}\right] \end{aligned} \tag{40}$$

$$I_{e2} = -\frac{1}{2\pi}\frac{I_S}{V_S - V_T}\left[V_{be}a\cos\left(\frac{V_T}{V_{be}}\right) - V_T\sqrt{1-\left(\frac{V_T}{V_{be}}\right)^2}\right] \tag{41}$$

$$Y_{T2} = \frac{I_{e2}}{V_b} = -\frac{1}{2\pi V_b}\frac{I_S}{V_S - V_T}\left[V_{be}a\cos\left(\frac{V_T}{V_{be}}\right) - V_T\sqrt{1-\left(\frac{V_T}{V_{be}}\right)^2}\right] \tag{42}$$

c.

$$\begin{aligned} &\int_{-\varphi_3}^{-\varphi_2} I_S \frac{V_{be}\cos\omega_0 t - V_T}{V_S - V_T}\cos\omega_0 t d\omega_0 t + \int_{\varphi_2}^{\varphi_3} I_S \frac{V_{be}\cos\omega_0 t - V_T}{V_S - V_T}\cos\omega_0 t d\omega_0 t = \\ &= 2\int_{\varphi_2}^{\varphi_3} I_S \frac{V_{be}\cos\omega_0 t - V_T}{V_S - V_T}\cos\omega_0 t d\omega_0 t = \\ &= 2\frac{I_S}{V_S - V_T}\left[\frac{V_{be}}{2}\omega_0 t + \frac{V_{be}}{4}\sin 2\omega_0 t - V_T\sin\omega_0 t\right]_{\varphi_2}^{\varphi_3} = \\ &= \frac{I_S}{V_S - V_T}\left[\begin{matrix} V_{be}\begin{pmatrix} a\cos\left(\frac{V_T}{V_{be}}\right) - a\cos\left(\frac{V_S}{V_{be}}\right) + \\ + \frac{V_T}{V_{be}}\sqrt{1-\left(\frac{V_T}{V_{be}}\right)^2} - \frac{V_S}{V_{be}}\sqrt{1-\left(\frac{V_S}{V_{be}}\right)^2} \end{pmatrix} - \\ -2V_T\left(\sqrt{1-\left(\frac{V_T}{V_{be}}\right)^2} - \sqrt{1-\left(\frac{V_S}{V_{be}}\right)^2}\right) \end{matrix}\right] = \\ &= \frac{I_S}{V_S - V_T}\left[\begin{matrix} V_{be}\left(a\cos\left(\frac{V_T}{V_{be}}\right) - a\cos\left(\frac{V_S}{V_{be}}\right)\right) - \\ -V_T\sqrt{1-\left(\frac{V_T}{V_{be}}\right)^2} - (V_S - 2V_T)\sqrt{1-\left(\frac{V_S}{V_{be}}\right)^2} \end{matrix}\right] \end{aligned} \tag{43}$$

$$\int_{-\varphi_2}^{\varphi_2} I_S\cos\omega_0 t d\omega_0 t = I_S\left[\sin\omega_0 t\right]_{-\varphi_2}^{\varphi_2} = 2I_S\sqrt{1-\left(\frac{V_S}{V_{be}}\right)^2} \tag{44}$$

$$\begin{aligned} &I_{e2} = \\ &= -\frac{1}{2\pi}\left[\frac{I_S}{V_S - V_T}\begin{pmatrix} V_{be}\left(a\cos\left(\frac{V_T}{V_{be}}\right) - a\cos\left(\frac{V_S}{V_{be}}\right)\right) - \\ -V_T\sqrt{1-\left(\frac{V_T}{V_{be}}\right)^2} - \\ -(V_S - 2V_T)\sqrt{1-\left(\frac{V_S}{V_{be}}\right)^2} \end{pmatrix} + 2I_S\sqrt{1-\left(\frac{V_S}{V_{be}}\right)^2}\right] \end{aligned} \tag{45}$$

$$Y_{T2} = \frac{I_{e2}}{V_b} =$$

$$= -\frac{1}{2\pi V_b}\left[\frac{I_S}{V_S - V_T}\left(\begin{array}{l} V_{be}\left(a\cos\left(\frac{V_T}{V_{be}}\right) - a\cos\left(\frac{V_S}{V_{be}}\right)\right) - \\ -V_T\sqrt{1-\left(\frac{V_T}{V_{be}}\right)^2} - \\ -\left(V_S - 2V_T\right)\sqrt{1-\left(\frac{V_S}{V_{be}}\right)^2} \end{array}\right) + 2I_S\sqrt{1-\left(\frac{V_S}{V_{be}}\right)^2}\right] \tag{46}$$

Now substitution of (31,32) and (42,46) into (27,28) results in the two-port admittance describing functions of the transistor model:

$$Y_I(|V_b|,|V_e|,\phi) = Y_{I1}(|V_b|,|V_e|,\phi) = \frac{I_E}{BV_b} I_1(V_{be} / nV_{TBE}) \tag{47}$$

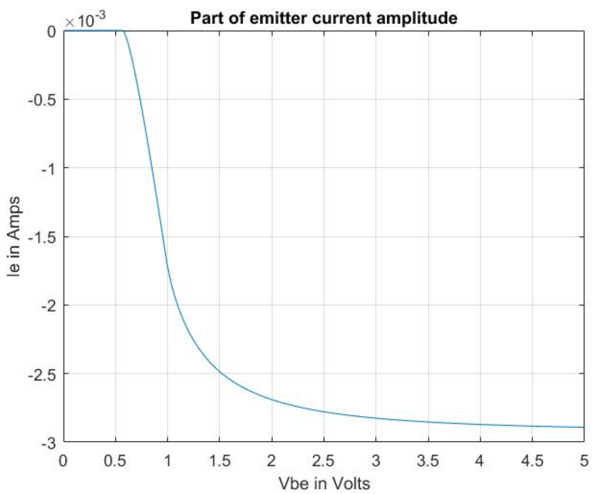

Figure 4. Emitter current amplitude as a function of the base voltage amplitude. Negative current is a consequence of the reference directions

$$Y_{T2}(|V_b|,|V_e|,\varphi) =$$

$$= \begin{cases} 0 \quad if \quad V_{be} < V_T \\ -\frac{1}{2\pi V_b}\frac{I_S}{V_S - V_T}\left[V_{be} a\cos\left(\frac{V_T}{V_{be}}\right) - V_T\sqrt{1-\left(\frac{V_T}{V_{be}}\right)^2}\right] \quad if \quad V_T \le V_{be} < V_S \\ -\frac{1}{2\pi V_b}\left[\begin{array}{l} \frac{I_S}{V_S - V_T}\left(\begin{array}{l} V_{be}\left(a\cos\left(\frac{V_T}{V_{be}}\right) - a\cos\left(\frac{V_S}{V_{be}}\right)\right) - V_T\sqrt{1-\left(\frac{V_T}{V_{be}}\right)^2} - \\ -\left(V_S - 2V_T\right)\sqrt{1-\left(\frac{V_S}{V_{be}}\right)^2} \end{array}\right) + \\ +2I_S a\cos\left(\frac{V_S}{V_{be}}\right) \end{array}\right] \quad if \quad V_S \le V_{be} \end{cases} \tag{48}$$

where we choose V_{be} as real. We show part of the emitter current amplitude coming from the nonlinear transconductance in the next Figure, with parameters I_S=9.2mA, V_T=0.57V and V_S=0.99V.

Plot in Fig. 4 has been generated by the following Matlab program (transfer_descr_fn1.m):

```
IS=9.2e-3;
VT=.570;
VS=.990;
Vmin=0;
Vmax=5;
N=1000;
Vstep=(Vmax-Vmin)/N;
Ie(1:N)=0;
Vbe(1:N)=Vmin+(1:N).*Vstep;

for i=(1:N)
    if Vbe(i)<VT
        Ie(i)=0;
    end
    if VT<=Vbe(i) & Vbe(i)<VS
        Ie(i)=-1/2/pi*IS/(VS-
VT)*((Vbe(i)*acos(VT/Vbe(i))-VT*sqrt(1-
(VT/Vbe(i))^2)));
    end
    if VS<=Vbe(i)
        Ie(i)=-1/2/pi*(IS/(VS-
VT)*(Vbe(i)*(acos(VT/Vbe(i))-
acos(VS/Vbe(i)))-VT*sqrt(1-
(VT/Vbe(i))^2)-(VS-2*VT)*sqrt(1-
(VS/Vbe(i))^2))+2*IS*sqrt(1-
(VS/Vbe(i))^2));
    end
end
plot(Vbe,Ie);
title('Part of emitter current
amplitude');
xlabel('Vbe in Volts');
ylabel('Ie in Amps');
grid on;
```

III. DESIGN OF A LOW FREQUENCY COLPITTS OSCILLATOR USING TWO-PORT DESCRIBING FUNCTIONS

We solve the design task by creating a small Matlab program that simulates the oscillator in frequency domain using the describing functions formulated above.

Circuit diagram of a Colpitts oscillator is shown in the next Figure.

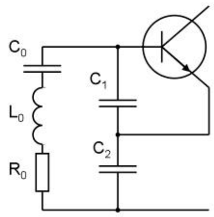

Figure 5. Circuit diagram of a Colpitts oscillator. DC bias elements are not shown

The Matlab file is the following (osc_sim4.m):

```
IS=9.2e-3; % [A]
VT=0.570; % [V]
VS=0.990; % [V]

C1=180e-12;  % [F]
C2=440e-12; % [F]
% L2=100e-6;
R2=470;
C0=47e-12; % [F]
L0=22e-6; % [H]
om0=1/sqrt(L0*1/(1/C0+1/C1+1/C2));
R0=13.721; % [Ohm]

nVTBE=0.02847;
ISBE=4.735e-13;
nVTBC=0.03399;
ISBC=1.67e-11;
IE=0.0053;
B=260;
VP=3;     %VP=20.5-2200*IE;
VB=nVTBE*log(IE/B/ISBE+1);

N=100;
Vb(1:N+1)=0;
Ve(1:N+1)=0;
Vbe(1:N+1)=0;
Ib(1:N)=0;
Ie(1:N)=0;
Y(1:2,1:2)=0;
Z(1:2,1:2)=0;

Y(1,1)=1/(1j*om0*L0+R0+1/(1j*om0*C0))+1j*
om0*C1;
Y(2,1)=-1j*om0*C1;
Y(1,2)=Y(2,1);
Y(2,2)=1j*om0*C2+1/R2+1j*om0*C1;

Z=inv(Y);

Vb(1)=0.3;
Ve(1)=0.1;

for i=(1:N)
Vbe(i)=abs(Vb(i)-Ve(i));
    if Vbe(i)<VT
        Ie(i)=0;
    end
    if VT<=Vbe(i) && Vbe(i)<VS
        Ie(i)=-1/2/pi*IS/(VS-
VT)*((Vbe(i)*acos(VT/Vbe(i))-VT*sqrt(1-
(VT/Vbe(i))^2)));
    end
    if VS<=Vbe(i)
        Ie(i)=-1/2/pi*(IS/(VS-
VT)*(Vbe(i)*(acos(VT/Vbe(i))-
acos(VS/Vbe(i)))-VT*sqrt(1-
(VT/Vbe(i))^2)-(VS-2*VT)*sqrt(1-
(VS/Vbe(i))^2))+2*IS*sqrt(1-
(VS/Vbe(i))^2));
    end

    Ib(i)=IE/B*besseli(1,(Vb(i)-
Ve(i))/nVTBE);
    Ie(i)=Ie(i)-IE/B*besseli(1,(Vb(i)-
Ve(i))/nVTBE);

Ib(i)=-Ib(i);
Ie(i)=-Ie(i);

    Vb(i+1)=Z(1,1)*Ib(i)+Z(1,2)*Ie(i);
    Ve(i+1)=Z(2,1)*Ib(i)+Z(2,2)*Ie(i);

end;

plot((0:N),abs(Vb),'r.',(0:N),abs(Ve),'b+
','LineWidth',1);
title('Oscillator voltage amplitudes');
ylabel('Vb(red), Ve(blue) in Volts');
xlabel('Iteration steps');
grid on;
```

The only problem is that it is very difficult to find proper initial values. Therefore we decided to implement the piecewise linear controlled source in AWR. Circuit details and results are shown in the next Figures.

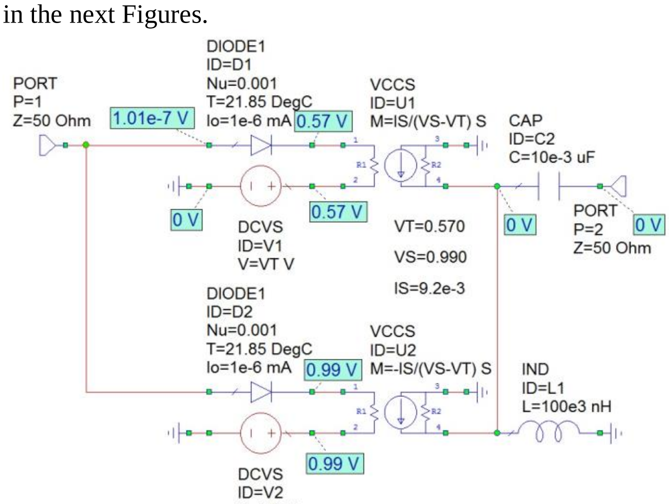

Figure 6. Circuit realization of the piecewise linear voltage controlled current source with characteristics shown in Fig. 3. Breakpoints are realized by diodes with high saturation current and

very low ideality factor. C2 prevents DC voltage at the output (first harmonic describing function), L1 leads DC current to the ground.

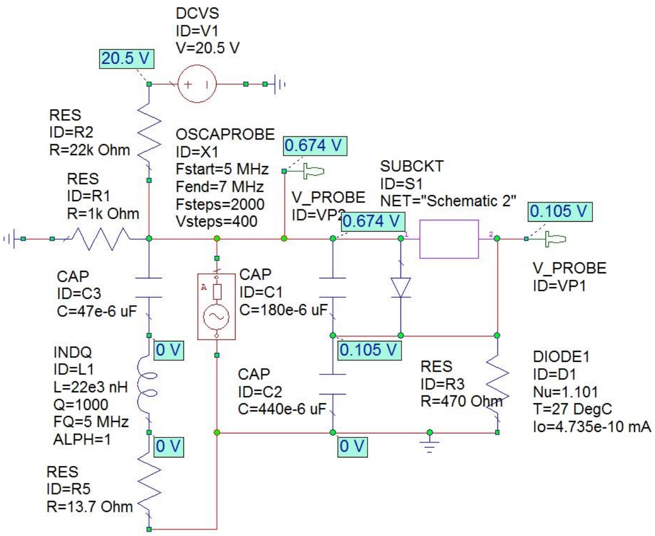

Figure 7. The oscillator circuit in AWR

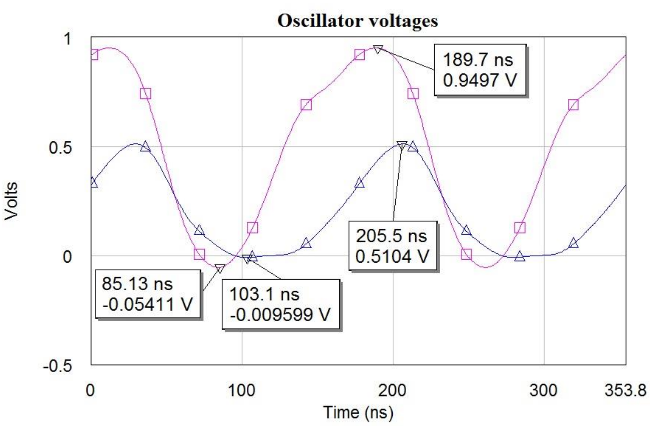

Figure 8. Oscillator voltages for the describing function method. Red: Vb, blue: Ve

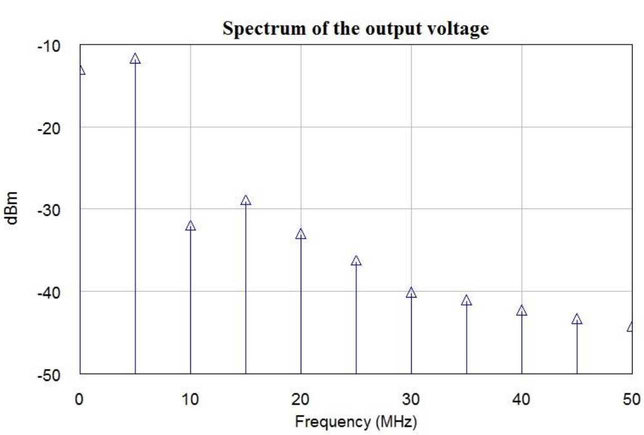

Figure 9. The output spectrum

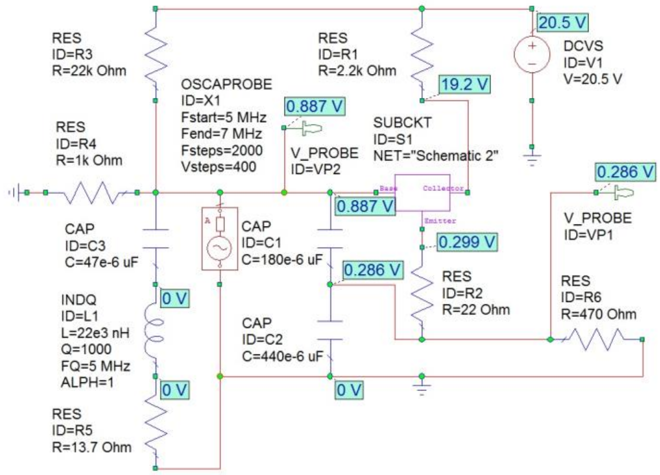

Figure 10. Circuit diagram

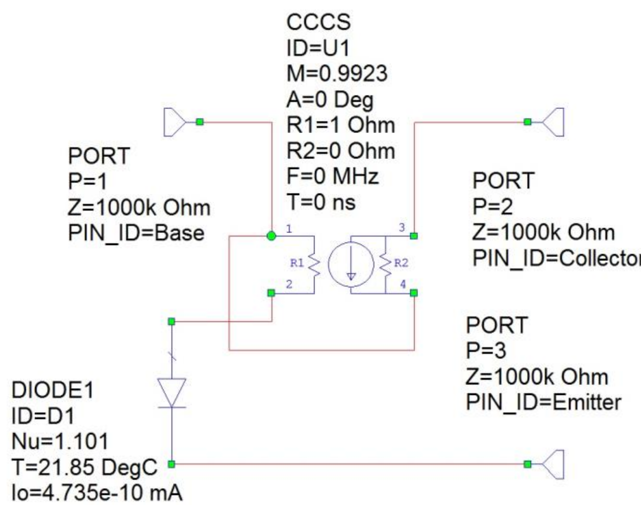

Figure 11. Transistor model

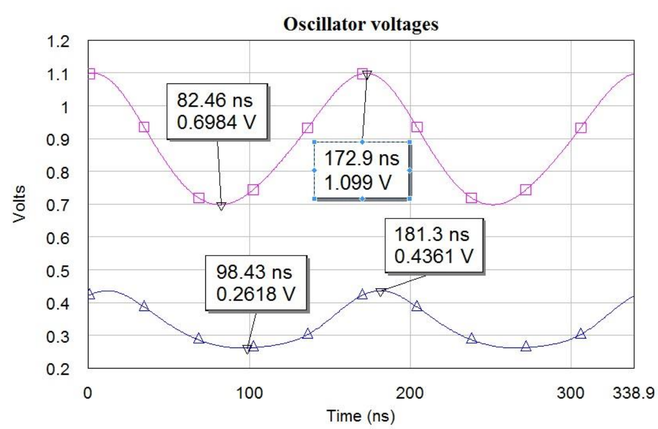

Figure 12. Oscillator voltages. Red: Vb, blue: Ve

Now we analyze the circuit in AWR. The circuit diagram is:

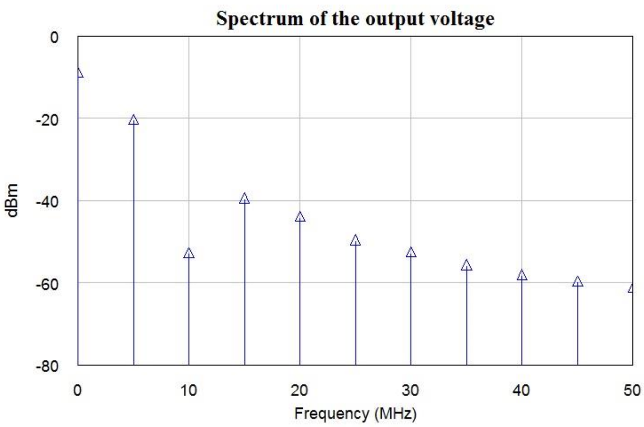

Figure 13. Spectrum of the output voltage. Please compare this Figure to Fig. 8

Table I. Comparison of data

	Describing function	AWR	Measurement
Vb (Vpp)	0.896 (Fig. 8)	0.401 (Fig. 12)	0.3594 (Fig. 15)
Ve(Vpp)	0.500 (Fig. 8)	0.175 (Fig. 12)	0.175 (Fig. 16)

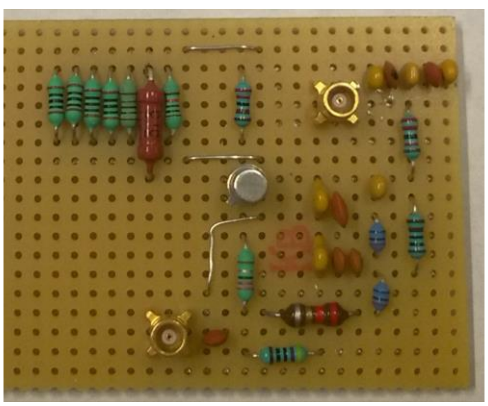

Figure 14. Photo of the oscillator realization

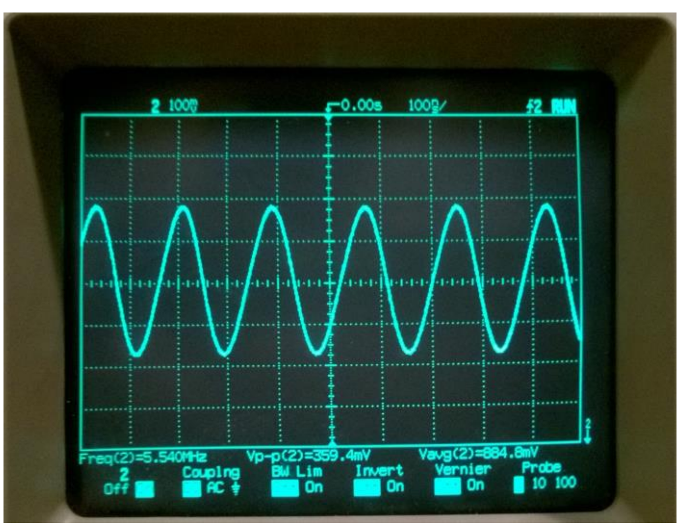

Figure 15. Photo of the base voltage measurement

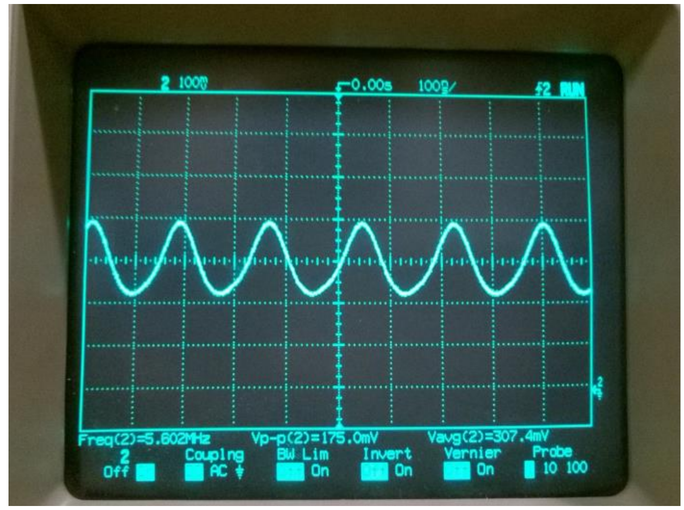

Figure 16. Photo of the emitter voltage measurement

IV. Conclusion

In this paper we studied oscillator design based on describing functions. A question may be why this procedure is better than a fully computerized one where oscillator parameters (frequency, power) are manually adjusted. In our opinion, the only right answer is that the method presented here gives a deep insight into circuit operation.

First we introduced two-port describing functions and their properties. Then the transformation between them has been derived. We used admittance describing functions for low frequency design.

There is a good coincidence between circuit analysis and measurements, and these results deviate from describing function method, see please Table I. This indicates a modelling problem that will be solved in our next publication.

Our plan is a series comprising three papers, this is the first one with introduction to the problem. In the second one we solve the accuracy problem. In the third one we present a microwave realization.

V. Acknowledgment

This work is intended as our modest commemoration of the 100th anniversary of Prof. K. Simonyi's birth.

Circuit building and measurements have been done at the Optical Laboratory of our University while paperwork has been done at Ericsson Telecom Hungary. Accordingly, Mr. V. Beskid and Mr. J. Benkő at Ericsson Telecom Hungary are acknowledged for the excellent research conditions. Also, we are grateful for Mr. T. Szili and Mr. K. M. Osbáth for their precise internal reviews.

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He was a member of the Telecommunication Systems Committee of the Hungarian Academy of Sciences (1985-97). He is an author or co-author of more than one hundred and fifty publications, including seven patents. He was a guest researcher in Helsinki (1993), Bologna (1994), Gothenburg (2010) and Stockholm (2013, 2014 and 2015).

Prof. Tibor Berceli made significant contributions in the field of microwave and optical technologies. His activity is concentrated on the nonlinear processes.

Prof. Berceli participated in many international research projects. Together with his group he contributed to the European ACCORD, Copernicus, MOIKIT, FRANS, LABELS and GANDALF projects. He participated in the European NEFERTITI and ISIS Network of Excellence and several COST co-operations.

Prof. Berceli is the author of 386 papers and 6 books published in English. He also has 26 patents. He was visiting professor at Polytechnic Institute of Brooklyn in 1964, University College London in 1986, Drexel University (Philadelphia) in 1988-89, Technical University of Hamburg-Harburg in 1991, Osaka University in 1992, Technical University of Grenoble in 1994, Helsinki University of Technology in 2001 and The Sydney University in 2004.