Attitude-driven Simultaneous Online Auctions for Parking Spaces

Levente Alekszejenkó and Tadeusz Dobrowiecki

***Abstract*—Among various parking assignment methods, auction-based procedures have emerged as a relatively simple and flexible mechanism to solve parking assignment and parking pricing problems. In the present contribution, we extend the usual purely financial bidder objective function with a mixed utility that also involves the walking distance to parking lots.**

We demonstrate numerous interesting properties of the new parking scheme; some of them are provable analytically; while others are traceable in simulation. At the end of the paper, we also present some practically useful examples.

***Index Terms*—auctions, parking, parking assignment, parking pricing, parking simulation**

I. Introduction

While cars offer convenient transportation, finding available parking is often difficult. Since Shoup's 2005 study, it has been commonly cited that 30% of urban traffic results from drivers searching for parking; though actual rates vary between 8–74% depending on location and time [1]. As parking occupancy nears 100%, search times can rise to 10–13 minutes [1].

An automated parking assignment system could reduce this inefficiency, saving time, easing congestion, and lowering environmental impact. With smartphones, cyber-physical systems, and IoT, deploying such a system (either as a new app or as part of a navigation tool) is feasible. Drivers could set their destination, parking budget, and walking preference, and be guided to a suitable spot. For instance, a Vickrey-Clarke-Groves (VCG) auction-based app was proposed in [2].

Beyond assigning parking spaces, auctions can dynamically adjust pricing based on demand. In 2011, San Francisco's *SFPark* initiative optimized parking fees using real-time occupancy data [3]. The program successfully maintained ideal occupancy levels by influencing behavior (encouraging long-term and budget-conscious drivers to choose cheaper, more distant spots). However, field studies [4], [5] show many drivers, especially on errands or commutes, are unwilling to forgo close parking. Preferences also vary by age, income, trip type, and number of passengers.

Motivated by these differing preferences, we propose a simultaneous online auction system that assigns and prices parking based on driver attitudes toward cost and walking distance. A smartphone app would automatically bid on behalf of users, reflecting these preferences. We demonstrate that this system aligns with user expectations through theoretical analysis and simulations using Eclipse SUMO [6].

To broaden the understanding of auction-based parking assignment, we present a short literature review in section II, an abstract parking supply and demand models are described in section III, and the handling of drivers' attitude is introduced in section IV. We formally describe *how this algorithm influences parking itself and what would be the experience of the stakeholders of the system* (e.g., drivers, municipalities, and parking lot operators) from a monetary (section V) and a monetary and distance related (section VI) point of view. Moreover, by explaining its implementation details in section VII, we also provide demonstrations for the theoretical results in section VIII, including an Eclipse SUMO-based simulation. After a short discussion of the applied method in section IX, section X concludes this paper.

II. Related Works

In 1969, parking lots occupied 87% of U.S. land-use coverage [7]. Today, European cities increasingly integrate parking policies into broader transport strategies to enhance accessibility, stimulate local economies, and improve quality of life. Common measures include limiting supply and using real-time dynamic pricing [8]. Criteria for evaluating such policies—like cost, walking time, capacity, and search time—are outlined in [9], which also classifies models by scale, from individual lots to city-wide systems.

For example, queueing models such as [10] analyze parking behavior within a single facility, while broader models like [11] examine city-wide impacts of pricing, congestion, and travel time, finding that real-time occupancy data can improve travel times by 4% and dynamic pricing can lead to socially optimal outcomes.

Dynamic pricing, first proposed in the 1950s, gained traction with large-scale implementations in the 2010s [12]. A wide range of techniques could be applied, including numerical optimization [13], Gale-Shapley algorithm-based matching techniques [14], [15]. Besides centralized approaches, distributed matching algorithms can also serve intelligent parking [16]. However, most of these solutions require additional steps to compute parking prices. The posted pricing algorithm also solves the parking charging problem [17].

Besides matching algorithms, dynamic programming, and game theory-based approaches [18] lead to auction-based systems. VCG auctions, originally designed for divisible goods with few participants [19], have been adapted for parking

L. Alekszejenkó and T. Dobrowiecki were collegues of the Department of Artificial Intelligence and Systems Engineering, Budapest University of Technology of Economics, Budapest, Hungary e-mail: {alelevente, dobrowiecki} (at) mit.bme.hu

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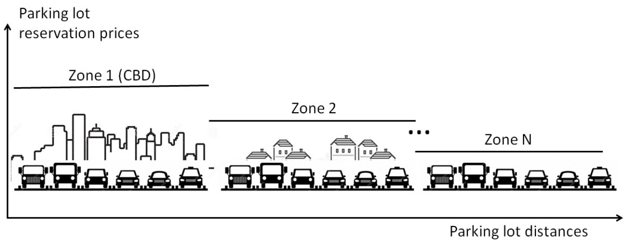

Fig. 1. Modeling parking in a city. Closer parking lots are more demanded and more expensive ones than further away alternatives.

allocation and pricing [20]–[23], although they often involve complex and time-consuming integer programming. To address this, [24] proposes more efficient alternatives.

In addition to VCG, ascending-bid English auctions are also viable. Bansal and Garg [25] introduce Simultaneous Independent Online Auctions (SIA), with Local Greedy Bidding (LGB). This LGB strategy ensures a bidder only bids on the items maximizing its utility, with auction prices increasing incrementally by ϵ.

Literature on parking mechanisms often emphasizes theoretical guarantees, such as individual rationality, budget balance, incentive compatibility, and asymptotic efficiency [20]–[23]. However, in-depth analysis of how these systems affect the stakeholders (e.g., drivers or parking operators) are rarely carried out. For example, [23] addresses user privacy, [18] discusses pricing-related properties, and [22] demonstrates expected utilities.

In this paper, we use the SIA/LGB approach of [25] to implement a solution similar to that in [26], in which all parking lot operators can organize an auction to sell their free parking spaces. Parking operators can set predefined starting prices[1] to be able to define a minimum parking costs, i.e., corresponding to traditional parking fees. The auctions shall run simultaneously, and they can increase actual bids by a predefined ϵ amount. With an attitude factor, drivers can express whether they prefer cheaper or closer parking lots. Our main contribution is the analytical and numerical analysis, including an Eclipse SUMO [6] traffic simulation-based demonstration of the SIA-based parking assignment and pricing system.

III. Parking Model

This paper uses a simplified model of parking supply and demand, assuming negligible differences in driving times, no traffic congestion or road tolls. Parking costs and walking distance are considered the primary factors influencing parking choice.

Parking demand is shaped by human activity and peaks at predictable times, such as just before working hours or some special events. We assume this demand is concentrated in the central business district (CBD), following some probability distribution.

[1]Starting prices are usually referred as reserve prices [27], but in this paper, we will use the term 'starting prices' to avoid confusion.

People vary in their tolerance for walking. Some, like plumbers or delivery drivers, require parking directly at their destination. Others, such as leisure visitors, may accept longer walks in exchange for lower fees.

Supply mirrors this pattern: parking prices are highest in the CBD and decrease toward the outskirts. Despite equal aerial distances, perceived walking distances can differ due to obstacles like rivers or railways, which help define parking zones [7]. These zones often reflect the same gradient in parking prices, see Fig. 1.

We propose an auction mechanism to assign parking spots based on drivers' preferences for cost and walking distance. In the following, we assume that there is an idealistic SIA with LGB algorithm-based application through which the drivers can reserve parking lots. This idealistic application has perfect information of free parking spaces, it is tamper-proof, and strictly follows the protocols of the proposed auction scheme. Additionally, drivers also obey the rules and occupy exactly those parking spots that the SIA-based algorithm assigns to them. For this study, we also assume that the parking supply and demand (including drivers' attitudes) are externally defined.

IV. Parking Assignment with Simultaneous Independent Online Auctions

In classical auctions, surplus has a monetary definition; however, when we use SIAs for parking assignment, the monetary surplus definition might not consider a natural behavior of drivers. Traditionally, when drivers arrive at their destination, they start cruising around until they can find a suitable parking place. As drivers generally turn [28], or generally drive in circles [29], trying to minimize the walking distance between the parking space and their destination. On the other hand, it is rational that one would like to minimize its parking expenses.

In traditional parking searches, it doubles the challenge to optimize both parking fees and walking distances. However, in an automated parking assignment method, we can expect to solve (at least partially) these optimization problems. Therefore, in this paper, we define a suitable preference function for the SIA with LGB strategy fusing monetary costs with walking distances to the parking lots.

Considering that there are $N \in \mathbb{N}^+$ parking lots in the area and each driver j, $j \in \{1, 2, \ldots, M\}, M \in \mathbb{N}^+$ has a limited monetary parking budget p_{v_j} corresponding to the *valuation* concept of classical auctions. Moreover, a driver agent j can compute the $d_{j,i}$ walking distance between the ith parking lot and its destination, where $i \in \{1, 2, 3, \ldots N\}$. Let p_i denote the actual parking cost at the ith parking lot, and let $d_{j,\max} = \max_i d_{j,i}$. Reflecting the attitude towards walking distance and parking prices, we propose to define a utility function $U_j(i)$ of each vehicle j for a parking lot i that is a combination of a monetary $0 \leq U_{p,j}(i) \leq 1$ and a distance-related $0 \leq U_{d,j}(i) \leq 1$ utility component corresponding to the state of the auctions:

$$U_j(i) = \beta_j U_{p,j}(i) + (1 - \beta_j) U_{d,j}(i), \tag{1}$$

using an attitude factor $\beta_j, 0 < \beta_j \leq 1$. By setting a low β_j, for example, a furniture deliveryman can prefer the closest parking lots. On the other hand, if we plan some leisure activity where walking *is* desired, we can set a high β_j and leave our vehicle in more distant, yet cheaper parking lots.

The monetary utility component can be: $U_{p,j}(i) = \frac{p_{\max} - p_i}{p_{\max}}$, where $p_{\max} = \max_i p_i$ corresponds to the maximum price achievable in auctions (e.g., the maximum valuation that drivers can afford). Similarly, the distance-related utility component can be: $U_{d,j}(i) = \frac{d_{j,\max} - d_{j,i}}{d_{j,\max}}$, where $d_{j,\max}$ is the distance between the most remote parking lot from the jth driver's destination.

Substituting these utility components with (1), we get the $U_j(i)$ utility function:

$$U_j(i) = \beta_j \frac{p_{\ max} - p_i}{p_{\max}} + (1 - \beta_j) \frac{d_{j,\max} - d_{j,i}}{d_{j,\max}} = \quad (2)$$

$$= 1 - \beta_j \frac{p_i}{p_{\max}} - (1 - \beta_j) \frac{d_{j,i}}{d_{j,\max}} = 1 - c_{j,i}, \quad (3)$$

where $c_{j,i}$ is the overall cost of the jth vehicle parking at the ith parking lot.

Therefore, we can define the Π_j auction with highest utility for the LGB strategy as:

$$\Pi_j = \arg\max_i U_j(i) = \arg\min_i c_{j,i}. \quad (4)$$

Naturally, since the j th driver's parking budget is limited by p_{v_j}, they shall only bid for the Π_jth parking lot, if $p_{\Pi_j} \leq p_{v_j}$. Consequently, at each moment, driver agent j can select a Π_j parking lot that appears to be suitable for bidding. As there could be multiple parking lot auctions that fulfill the criterion of (4), in the following, we will treat Π_j as a set of appropriate parking auctions. The following Theorem proves that the proposed algorithm will stop after receiving a sufficient amount of bids.

Theorem 1. *If $\beta_j > 0$ and $p_{v_j} > 0$ are finite (individual) constants for all bidders, then the SIA algorithm will terminate.*

Proof. The SIA method terminates if and only if it finds a good assignment. As current prices increase by ϵ upon receiving a new bid, sooner or later the current prices will either reach a state that is a solution or they reach the p_{v_j} valuation of the bidders. Denoting $p_{v_{\max}} = \max_j p_{v_j}$ and $p_{\min}^{(0)} = \min_i p_i^{(0)}$, where $p_i^{(0)}$ stands for the starting price of auction i, the $t_{\max}$ number of bids to reach the valuation of all bidders in all N auctions is bounded by:

$$t_{\max} = \left\lceil \frac{p_{v_{\max}} - p_{\min}^{(0)}}{\epsilon} \right\rceil N. \quad (5)$$

Consequently, the SIA method terminates after receiving $t \leq t_{\max}$ bids. □

In the following, to analyze the proposed system, we demonstrate some theoretically provable properties, illustrate them by numerical simulations, and also provide some application use cases.

V. Distance-independent Properties of the Parking SIA

Now, we will examine the SIA mechanism for parking lot assignment with the utility function defined in (2), analytically if it is possible or by simulations. By varying the β_j attitude factor, we favor smaller walking distances or lower parking costs.

If a driver chooses an $\beta_j \approx 0.0$ value, it results in a classical parking search algorithm, when people want to park near their destination, regardless of parking fees. In this case, the SIA algorithm is only expected to provide an assignment between parking lots and vehicles without optimizing parking costs, which in over-demand situations ($M > N$) can reach p_{v_j}.

On the other hand, $\beta_j \approx 1.0$ can also be a relevant selection if there are multiple parking lots, possibly having different parking prices, in such a small area, in which drivers will not perceive significant differences in walking distance. Supposing a small area and an $\beta_j = 1.0$ setting, the SIA method can purely optimize parking costs in addition to assigning vehicles and parking spaces. The following lemmas and theorems help us to understand the effect of SIAs on parking prices.

A. Expected Parking Prices

First of all, let us check how prices change during the execution of SIA. Let $\mathbf{p}^{(t)}$ parking prices vector collect all the actual parking prices $p_i^{(t)}$ that received bids after t steps of auctions. Then, Lemma 1. expresses the price changes in SIA.

Lemma 1. *$\exists T$, such that after running SIA for $t > T$ steps, the difference between the elements of the current price vector $\mathbf{p}^{(t)}$ will be $\leq \epsilon$.*

Proof. The proof is given in Appendix A. □

Naturally, the rate of supply and demand for parking lots define parking costs in market-driven pricing scheme. As SIA reacts to the actual parking situation, we can expect that the obtained parking prices obtained also reflect it. Theorem 2. refers to the situation in which the supply is either in equilibrium with the demand or exceeds it.

For a compact notation, let us define two symbols. Firstly, let us create an *ascending list* of starting prices of the auctions. Here, $p_{M^-}^{(0)}$ will denote the Mth element of the list, corresponding to the Mth *lowest* starting price. Secondly, we create a *descending list* of valuations of the bidders. Here, p_{v,N^+} will denote the Nth element of the list, corresponding to the Nth *highest* valuation.

Theorem 2. *Assuming that there is no over-demand for parking ($N \geq M$) and $\beta_j = 1.0$, each element of the assigned, won parking prices vector $\mathbf{p}$, $\mathbf{p} \in \mathbb{R}^M, M > 1$ provided by the SIA method will be approximately equal to the Mth lowest $p_{M^-}^{(0)}$ starting price: $p_{M^-}^{(0)} \leq p_i \leq p_{M^-}^{(0)} + \epsilon < p_{v_j}$ for $p_i \in \mathbf{p}, j \in \{1, 2, \ldots, M\}$.*

Proof. With the assumptions, the utility function of each j driver agent simplifies to $\Pi_j^{(t)} = \arg\max_i \left(1 - \frac{p_i^{(t)}}{p_{\max}}\right)$ in the tth step of the auction. Consequently, following the LGB strategy, each j driver will bid for one of the cheapest parking lots. Moreover, $p_i^{(t+1)} = p_i^{(t)}$ or $p_i^{(t+1)} = p_i^{(t)} + \epsilon$, depending on whether or not someone has bid for it.

If the starting prices are identical, $p_i^{(0)} = p_{i+1}^{(0)}, \forall i : i \in \{1, 2, \dots N-1\}$, and there is no over-demand, the statement is a natural consequence, and the resulting prices will be the original starting prices.

If the starting prices are different, we shall prove that the M vehicles will compete for the cheapest M parking lots until the resulting prices will not differ more than ϵ, and each vehicle will have been winning exactly one parking space auction. Lemma 1. proves this case.

□

On the other hand, the demand can also exceed the supply of parking lots. Theorem 3. shows what would happen in this scenario when we use the SIA algorithm.

Theorem 3. *Assuming that there is an over-demand for parking ($N < M$) and $\beta_j = 1.0$, each element of the winning parking prices vector* $\mathbf{p}$, $\mathbf{p} \in \mathbb{R}^N, N > 1$ *provided by the SIA method will be approximately equal to the $(N+1)$th highest $p_{v,(N+1)^+}$ valuation: $p_{v,(N+1)^+} \le p_i \le p_{v,(N+1)^+} + \epsilon$ for $i \in \{1, 2, \dots, N\}, j \in \{1, 2, \dots, M\}$.*

Proof. The proof is given in Appendix B. □

In summary, due to Lemma 1., prices increase rationally. In a non-over-demand scenario, this process leads to an assignment on the price of the Mth cheapest parking lot according to Theorem 2. In contrast with this, in an over-demand scenario, the richest N drivers (those who have the highest p_{v_j} valuations) can find parking spaces for themselves because of Theorem 3.

B. Optimal Parking Lot Size

From another perspective, parking lot operators might face the problem of having to decide how many parking spaces are required to maximize their revenue. The following corollary helps find the optimal parking lot size to maximize parking incomes if the parking pricing system is driven by SIA with the LGB strategy.

Corollary 3.1. *Taking into account the constant demand of $M > 2$ vehicles with $\beta_j = 1.0$, $p_{v_j} > p_i^{(0)}$ for $i \in \{1, 2, \dots, N\}, j \in \{1, 2, \dots M\}$, an optimal parking lot, which maximizes parking incomes by running SIA with the LGB strategy, provides $N = M - 1$ parking spaces.*

Proof. Let $Np_v = \sum_{j=1}^{N} p_{v,j}$. Then, in the over-demanded ($N < M$) case, the operator can obtain an income of Np_v, while in the $N \ge M$ case, it can earn $Np_i^{(0)}$. As $p_v > p_i^{(0)}$, at the $N = M$ setting ($Np_v > Np_0^{(0)}$), the operator would lose a significant amount of money.

As Np_v is a strict monotonic function of N, let us check whether the total income in the $N = M - 1$ exceeds the $N = M$ setting:

$$(M-1)p_v >_? Mp_i^{(0)} \tag{6}$$

$$M >_? 1 + \frac{p_i^{(0)}}{p_v - p_i^{(0)}} \tag{7}$$

As $p_v > p_i^{(0)}$, the right hand side converges to 1.0; hence, if $M > 2$, then the statement is true. □

VI. DISTANCE-DEPENDENT PROPERTIES OF THE PARKING SIA

To make driver decisions more flexible, we introduced a mixed cost and distance-aware utility (2) in section IV. We anticipate a more cost-aware driver (who bids with a high attitude factor β_j) would win a cheaper yet more distant parking lot, and vice versa.

As drivers bids for parking lots maximizing their momentary utility, the following lemma clarifies the relation between the maximum utility and the driver's awareness of costs over distances.

Lemma 2. *Assuming that two parking lots i and i' are equally useful for a vehicle j, at some β_j: $U_j(i) = U_j(i')$. Without loss of generality, let us also assume that $p_i > p_{i'}$ and $d_{j,i} < d_{j,i'}$.*

Then, by changing β_j with $\Delta_\beta > 0$, vehicle j will prefer either parking lot i or i' as follows:

(I) If $\beta_1 = \beta_j - \Delta_\beta$, then vehicle j will prefer the closer but more expensive parking lot, i.e.: $U'_j(i) > U'_j(i')$.

(II) If $\beta_2 = \beta_j + \Delta_\beta$, then vehicle j will prefer the more distant but cheaper parking lot, i.e.: $U'_j(i) < U'_j(i')$.

Proof. We check the two cases:

(I) If $\beta_1 = \beta_j - \Delta_\beta$ then $U'_j(i) > U'_j(i')$.

$$U'_j(i) >_? U'_j(i') \tag{8}$$

$$U_j(i) + \Delta_\beta\left(\frac{p_i}{p_{\max}} - \frac{d_{j,i}}{d_{j,\max}}\right) >_? \tag{9}$$

$$>_? U_j(i') + \Delta_\beta\left(\frac{p_{i'}}{p_{\max}} - \frac{d_{j,i'}}{d_{j,\max}}\right) \tag{10}$$

Following the assumption $U_j(i) = U_j(i')$:

$$0 >_? \Delta_\beta\left(\frac{p_{i'} - p_i}{p_{max}} + \frac{d_{j,i} - d_{j,i'}}{d_{j,\max}}\right) \tag{11}$$

$p_i > p_{i'}$ and $d_{j,i} < d_{j,i'}$ yield that the right-hand side of the above expression is negative. Hence, the first statement is true.

(II) If $\beta_2 = \beta_j + \Delta_\beta$ then $U'_j(i) < U'_j(i')$. Analogously to (I), and following the assumption $U_j(i) = U_j(i')$, we can get:

$$0 <_? \Delta_\beta\left(\frac{d_{j,i'} - d_{j,i}}{d_{j,\max}} + \frac{p_i - p_{i'}}{p_{max}}\right) \tag{12}$$

As $p_i > p_{i'}$ and $d_{j,i} < d_{j,i'}$, the right-hand side of the above expression is positive. Hence, the second statement is also true.

□

Lemma 2 only presents an elementary step of the bidding process. Following analytically the distribution of the winners' attitudes is perhaps inextricable considering the parallel bidding and the randomness of the driver attitudes. However, we expect that a certain overall regularity might emerge (on the average) in the bidding process.

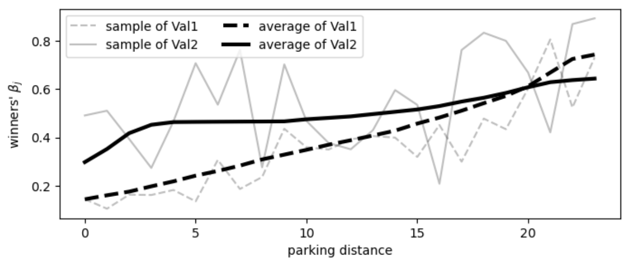

Fig. 2. Distribution of winners' attitudes in function of distance of the won parking lots.

To illustrate it, we present in Fig. 2. the attitude distribution among the winner drivers as a function of the parking lot distance, for an individual auction (`sample`), and for the auction results averaged over 10000 auction runs (`average`). In the simulation (see section VII), $M = 35$ drivers competed for $N = 24$ parking lots with the same starting price ($\forall i : p_i^{(0)} = p_{r,const}$), evenly spaced with increasing distance from the center. The drivers' attitudes were sampled from a uniform distribution $\beta_j \sim \mathcal{U}(0.1, 0.9)$. The drivers' validations were constant i.e.: $\forall j : p_{v_j} = V$ in the `Val1` scenario; and in `Val2`, they were sampled from a uniform distribution $p_{v_j} \sim \mathcal{U}(\frac{2}{3}V, V)$ where V was the maximal validation and $\frac{2}{3}V > p_{r,const}$.

We can observe in Fig. 2. that `sample` runs are increasing erratically; however, the expected values are smoothly monotonic. Indeed, with the introduced utility mechanism, drivers who opt for cheaper but more distant parking lots win on the average accordingly, and in a distance-dependent proportional way. The difference in behavior of the two cases is most certainly due to the uncorrelated character of random validations (`Val2`) and the random attitudes. When a strong negative correlation is introduced between random validation and random attitudes (i.e., the higher validation pairs with a lower attitude; those who have more money want to park closer), the `sample` and `average` curves are similar to the `Val1` case.

VII. On the Implementation of Parking SIA

Originally, SIA algorithm can be executed in a highly parallel way, in which each parking space can run its own auction server and all parking-seeking drivers (or automated bidder agents impersonating them) can send bids asynchronously. Unfortunately, simulating this parallelism on the PC-based research architecture is not efficient because of the frequent context changes during parallel execution.

Consequently, we serialize the execution of the SIA algorithm. It requires a slight modification of the original behavior of the bidders. Instead of asynchronous execution, the bidder will execute an event-driven program, following the state machine in Fig. 3, of which more complex functions are described by Algorithm 1.

Upon request from a particular auction a_i, a bidder computes whether it is willing to bid for it or not following Algorithm 1. In parallel execution, this function would be

Algorithm 1 Bidders' main functions

function BIDDER.ASK_BID(a_i)
  $bids \leftarrow$ False
  recall $state$
  recall $\mathbf{p}$ ▷ all price values
  $\mathcal{F} \leftarrow \{a_j : p_j \leq p_v, j \in \{1, 2, \ldots, N\}\}$ ▷ feasible a_j's
  $\Pi = \arg\min_k c_k$ ▷ computing preferences as in (4)
  if ($state =$ 'overbid') $\wedge$ ($p_i \leq p_v$) **then**
    $\mathcal{P} \leftarrow \{a_j : a_j \in \mathcal{F} \bigcap \Pi\}$ ▷ preferred auctions
    $bids \leftarrow (a_i \in \mathcal{P})$
    if $bids$ **then**
      store $state \leftarrow$ 'winning'
    end if
  end if
  return $bids$
end function
function BIDDER.INFORM_PRICE(p_i)
  store p_i
  recall $state$
  if ($\nexists j : p_j \leq p_v$) $\wedge$ ($state \neq$ 'winning') **then**
    store $state \leftarrow$ 'out of budget'
  end if
end function

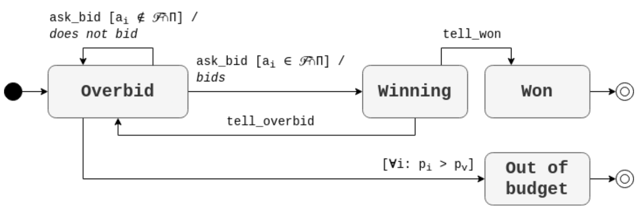

Fig. 3. States machine representing the `Bidder` agent.

similar; the only difference is that some scheduler algorithm would call this subroutine, and it would return a random element from the $\mathcal{F} \cap \Pi$ set to bid for.

The main difference between SIAs and the serialized simulation is described in detail in Algorithm 2. Instead of running many algorithms in parallel, we iterate over them and over all the bidder agents in line 6 and line 7. The code inside these iterations executes the event-driven calculations.

As stated above, we modified SIA in numerous points to be able to simulate it on simple PCs. Theorem 4. justifies that a fully parallel SIA is the generalization of the serialized auction implementation. Consequently, all theorems, lemmas, and corollaries of sections IV, V, and VI are also necessarily true for serialized auctions.

Theorem 4. *The fully parallel execution of the SIA is a generalization of the serial execution of the auction method.*

Proof. The proof is given in Appendix C. □

VIII. Examples

By running the serialized SIA defined in section VII, we made experiments to demonstrate some of the theorems, lemmas, and corollaries of section V and VI. Furthermore, we provide some application examples of the proposed method.

Algorithm 2 Auctioneers' algorithm

1: **function** RUN_AUCTIONS(A, B, $\mathbf{p}_s$, ϵ, $r_{\max}$)
2: $\mathbf{w} \leftarrow \emptyset$ ▷ init: no winner
3: $\mathbf{p} \leftarrow \mathbf{p}_s$ ▷ init: price = starting price
4: $B_j.state \leftarrow$ 'overbid' $\forall B_j \in B$ ▷ init: state of bidders
5: **while** $\exists l : B_l.state =$ 'overbid' **do** ▷ $\exists$running bidder
6: **for** $i \in \{1, 2, \ldots N\}$ **do** ▷ iterating on auctions
7: **for** $j \in \{1, 2, \ldots, M\}$ **do** ▷ iterating on bidders
8: $bids \leftarrow B_j$.ASK_BID(a_i)
9: **if** $bids$ **then**
10: **if** $w_i \neq \emptyset$ **then** ▷ inform previous winner
11: w_i.TELL_OVERBID
12: **end if**
13: $w_i \leftarrow B_j$ ▷ new winner
14: $p_i \leftarrow p_i + \epsilon$ ▷ increase price
15: **for** $k \in \{1, 2, \ldots, M\}$ **do**
16: B_k.INFORM_PRICE(p_i)
17: **end for**
18: **end if**
19: **end for**
20: **end for**
21: **end while**
22: **for** $i \in \{1, 2, \ldots N\}$ **do**
23: w_i.TELL_WON
24: **end for**
25: **return** $\mathbf{w}, \mathbf{p}$
26: **end function**

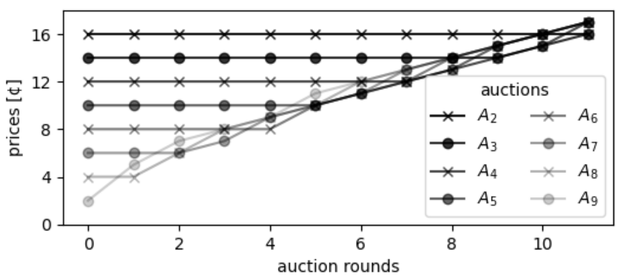

Fig. 4. Parking prices during auctions.

A. Illustration of Lemma 1

According to Lemma 1, prices will be approximately equal after a number of auction rounds. To demonstrate this, we created a simple simulation with $N = 10$ parking spaces and $M = 8$ vehicles. The starting prices at the corresponding $A_0, A_1, A_2, \ldots A_9$ auctions were $20, 18, 16, \ldots 2$ ¢. For a simple illustration, we assumed here that drivers do not differentiate parking spaces by distance. We used a bid step of $\epsilon = 1.0$ ¢, and the valuation of each $j \in \{1, 2, 3, \ldots 8\}$ vehicle was $p_{v_j} = 1000$ ¢.

Fig. 4. shows the resulting parking prices after each auction round. An auction round corresponds to the state achieved after a run of iteration of line 6 in Algorithm 2. The results show that the current prices at each of (the cheapest M) auctions increase together. The final winning prices are at most ϵ apart from each other.

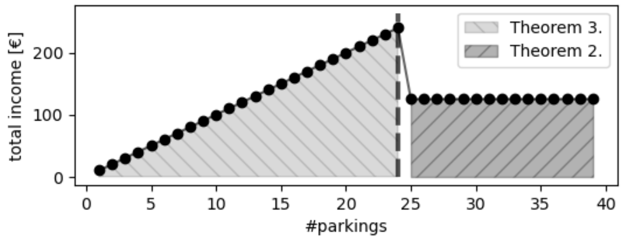

Fig. 5. Optimal sizing of a parking lot to maximize the operator's income at a demand of $M = 25$ vehicles.

B. Illustration of Corollary 3.1

As Corollary 3.1 combines the consequences of Theorem 2 and Theorem 3, we created a simulation to demonstrate them together. Hence, we assume a parking demand of $M = 25$ vehicles, and after running SIAs we computed the total income of the parking lot operator for various $N \in \{1, 2, 3, \ldots, 40\}$. The attitude factor was $\beta = 1.0$, the bid step was $\epsilon = 1.0$ ¢, and the valuation of each $j \in \{1, 2, 3, \ldots 8\}$ vehicle was $p_{v_j} = 1000$ ¢. The starting prices for each auction were 500 ¢.

On the left-hand side of Fig. 5., when $M < 25$, we can find the demonstration of Theorem 3 regarding the parking prices obtained in the over-demanded case. On the right-hand side of Fig. 5., when $M \geq 25$, we can observe the consequences of Theorem 2 regarding parking prices in a not over-demanded scenario. Finally, Fig. 5. also shows that the operator of the parking lot can maximize its income with an $N = M - 1 = 24$ setting, which aligns with Corollary 3.1.

C. Application: Offline Parking Pricing

The proposed auction mechanism is suitable to provide an intelligent parking pricing model in a city that calculates with a spatial (s) parking demand $D(s)$. The demand model $D(s)$ can originate from historical time series. By setting a lower attitude towards walking, e.g., $\beta = 0.1$, and using the $D(s)$ demand model, we can simulate the auction methods for a known number of vehicles (M) and parking lots (N).

To demonstrate that the auction method can suggest efficient parking price settings, we created an abstract simulation. As there are hundreds of cities in the European Union that have imposed low emission zones (LEZs), various restrictions on vehicles that can enter specific areas [30], we also model parking prices at the perimeter of an LEZ. We used $N = 474$ parking spaces in groups of 6 spaces, 50 m apart from each other. Between the positions of 3000 m and 4000 m, there was an LEZ where there were no parking lots.

From a cross section view of a city, the parking demand of $M = 200$ vehicles followed the mixed probability distribution of $D_1(s) \sim \mathcal{U}(0, 5000)|_{100} + \mathcal{N}(1000, 100)|_{100}$, where 100 vehicles had a uniform demand across the whole a city, and 100 vehicles were headed towards the 1000 m point, following a normal distribution with $\sigma = 100$ m. For all j vehicles, the valuation was $p_{v_j} = 1000$ ¢, and the bid step was $\epsilon = 1.0$ ¢in each auction. In the initial phase, there was free parking in the city. After running the SIA method, we got the results

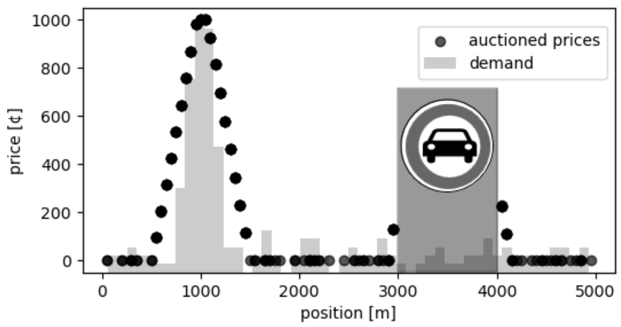

Fig. 6. SIA simulation results for a city containing a peak in the demand at the position of 1000 m, and a low emission zone between positions of 3000 m and 4000 m.

of Fig. 6., yielding that parking prices shall increase around the peak of demand, and at the perimeter of the LEZ, while parking could remain free in the rest of the city.

D. Application: Online SIA in a rural town's parking

Besides offline parking pricing optimization, SIAs might run in real time. For example, drivers might use their smartphones to navigate to a parking lot assigned to them by the SIA method, depending on their destination and attitude towards walking and parking prices. Hence, we have tested the SIA method in generated but realistic simulation of a typical European small rural town [31] having 10.000 inhabitants and commuters carrying out their daily activities. Consequently, this test is hardly a corner case for parking assignment, but there might be spatial and temporal over-demands in particular parts of the small town.

After 3 simulated days of bootstrapping, we simulated the morning traffic of the fourth day between 6:00 am and 10:00 am. During this simulation, we started SIAs for all empty parking spaces, and the simulated vehicles had to bid for them to reserve parking places, and to negotiate an hourly parking price. The starting prices for each auction were 50 ¢, and the bid step was $\epsilon = 5$ ¢. Each vehicle j had a valuation of $p_{v_j} = 1000$ ¢= 10 €. Originally, vehicles aimed to use the curbside parking lot, provided on both sides of each road segment, but at the most visited sites, we placed 8 parking garages, see Fig. 7. Parking garages are considered *alternative* parking solutions and had a fixed price of 1000 ¢. Consequently, if a vehicle could not get a curbside parking space on auctions, it would be able to use the nearest parking garage.

We experimented with different β_j settings. We tried 3 scenarios in which all vehicles had the same attitude β_j of $0.1, 0.5$ or 0.9. Furthermore, we simulated a `MIX` scenario with 90% of the vehicles having a $\beta = 0.1$ and 10% of the vehicles having a $\beta = 0.9$ attitude, representing that 90% of the drivers want to park near their destinations, but there is a minority, who prefers longer walking distances in favor of cheaper parking alternatives.

In addition to walking distances and auctioned hourly parking prices $p_{j,i}$, we also measured the duration of parking

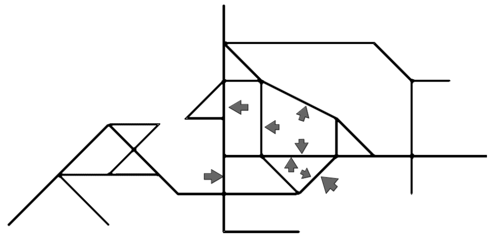

Fig. 7. Road network of the simulated town in Eclipse SUMO. Parking garages are indicated on their corresponding side of the roads.

TABLE I
ECLIPSE SUMO SIMULATION RESULTS (AVERAGE OF 5 RUNS)

β	total price [€]	distance [m]	auctioned price [€/h]	parking garage alternatives
0.1	4.86	13.81	51.97	3.28%
0.5	4.68	32.03	50.05	0.00%
0.9	4.68	32.02	50.00	0.00%
`MIX`	4.82	14.84	51.40	3.14%

$\tau_{j,i}$ (in seconds) to calculate the total parking price $P_{j,i}$ as: $P_{j,i} = p_{j,i} \cdot \left\lceil \frac{\tau_{j,i}}{60 \cdot 60} \right\rceil$.

Moreover, we tracked the ratio of taking the parking garage alternative to the number of all simulation participation.

Table I summarizes the average results obtained from 5 distinct simulation runs of all β attitude settings. As we expected, when drivers prefer closer parking lots, it decreases walking distances and increases parking prices. In approximately 3% of the cases in this particular scenario, low β attitude setting creates such an over-demand that some vehicles have to opt for parking in parking garage instead of curbside parking lots.

IX. DISCUSSION

In this paper, we assume that parking fees and walking distances are the primary determinants of the choice of parking lot. In reality, there are additional influencing factors, for example, road tolls, traffic congestion, and the availability of public transportation. From an abstract point of view, this extra information could be integrated into (1), and with proper modifications, one could design attitude factors towards these new factors as well.

Moreover, the survey of [32] indicates some usual performance metrics to evaluate a dynamic parking pricing scheme. In the following, we describe how the SIA/LGB for parking assignment performs according to the factors relevant for dynamic parking pricing.

As the proposed method provides parking assignment, it reduces *environmental factors*, eliminates *parking searching time*; therefore, it improves *average speed* and *traffic flow*. By properly setting the attitude factor β_j, drivers can optimize its *travel time* and *utility* corresponding to their *ridership* and actual *demand*. Considering drivers' attitudes, the proposed SIA-based solution provides a more flexible approach of parking assignment compared to a posted pricing-based method, in which walking distance is defined as the social cost [17].

Following the offline results of SIA/LGB as in section VIII-C, municipalities can optimize parking prices or, following Corollary 3.1, the number of available parking lots to optimize their *revenue* by balancing the *supply*. However, drivers might face higher parking *costs* in periods of overdemand. Operators can use this extra income to cover their *operational costs*. Furthermore, Corollary 3.1 can also help parking lot operators maximize their incomes by optimizing the number of offered parking spaces on the auction rounds. If they have a demand forecast for parking, they can offer fewer than all free parking spaces in an area in specific periods. For example, a municipality might not offer all free parking spaces in the early morning to be able to serve the parking demand during later rush hours.

In terms of *computational complexity*, a fully parallel implementation of SIA auctions can run in linear time, see Theorem 1., while bidders shall run in a second-order polynomial time of the number of auctions (they shall regularly check the state of all auctions and send bids if necessary). In section VIII-D, we also presented that in a usual European rural town, around 3% of the *users get rejected*, and shall look for an alternative parking solution. In this study, we demonstrated the core parking assignment method, assuming all the drivers are cooperative, obey the rules, and occupy exactly those parking spaces reserved for them by the auction method. However, in the real world, not all drivers will or can use the implementation of the proposed system. To this end, we plan further analysis with different rates of participating and non-participating drivers to evaluate the system's properties in mixed traffic. Moreover, it would also be a further research question how strategic bidding of a group of drivers would impede the proposed method.

We have also assumed that the method has perfect information about parking lot occupancies. We ran simulations to relax this assumption and found that relatively accurate measurements are necessary (at least 93% of accuracy). Fortunately, state-of-the-art camera-based solutions can achieve this level of accuracy [33].

In this study, we assumed a predefined parking demand. In reality, various factors can influence where drivers would like to park. For example, one is likely to avoid standing in traffic congestion and prefer a park-and-ride (P+R) parking facility with good public transportation connections. We can model this attitude in the proposed solution as a slight preference for higher walking distances over parking costs. Moreover, there could be road tolls along the way from the suburbs towards the CBD of a city. One might offset the starting prices of the inner parking spaces with these fees to model the overall costs. Drivers with limited transportation budgets or who prefer cheaper parking lots might avoid these expensive alternatives by properly setting their β_j attitude factor.

Actual traffic situations, such as unusually slow traffic, also influence the choice of parking lot. By adjusting the β_j attitude factor, the proposed method could react to the changed situation. Consequently, properly setting the attitude factor might be a challenging task. By applying reinforcement learning, we might implement an automated approach to set the attitude factor for each driver appropriately. However, it would require feedback from the drivers, which, for example, would be given by answering some questions (e.g., *Did you have to walk far?* or *Did you find the parking fees high?*).

X. CONCLUSION

In this paper we systematically analyzed how simultaneous online independent auctions with local greedy bidding strategy of [25] could be applied to the assignment of parking spaces. In our solution, similarly to [26], we used an attitude factor β_j that could be set by drivers to reflect their attitude towards closer and more expensive over more distant but cheaper parking spaces.

We theoretically proved that this method sooner-or-later terminates (Theorem 1). Additionally, we had propositions on how the prices are reaching each other and then increasing together during the auctions (Lemma 1), and what could be the price outcome of the auction method in non-overdemanded (Theorem 2, vehicles win parking spaces for the Nth lowest starting prices) and over-demanded situations (Theorem 3, richest M vehicles obtains parking lots for the valuation of the Mth richest drivers). Following those theorems, we provide a formula to calculate the optimal parking size $(M-1)$, if the incoming parking demand (M) is known (Corollary 3.1). Moreover, we showed that different attitude factors *have* an effect on the obtained parking lots (preferring cheaper parking lots results in more distant parking), see Theorem 2 and section VI.

By implementing SIAs in a PC environment, we demonstrated in section VIII-C how a municipality can use offline results of the auction method to implement demand-related parking prices, or how can an online deployment conduct the parking assignment in a simulated rural town, see section VIII-D.

With modern smartphones and mobile connection, there is no longer an obstacle to implement an auction-based parking guidance, assignment, and pricing system. We hope that our results add to the vision of a modern city's parking system that has the potential to make parking more efficient, and lead to urban redevelopment, leaving more space to parks, sustainable micromobility-based transportation, and local businesses, to make our habitat more livable. To encourage further experimentation and facilitate further studies, we provide our source codes here: https://github.com/alelevente/auction_theories.

APPENDIX A
PROOF OF LEMMA 1.

Proof. We prove the statement by induction.

Without loss of generality, consider a simple scenario with two parking lots (A_1, A_2), two vehicles (B_1, B_2), and $p_{A_1}^{(0)} > p_{A_2}^{(0)}$. Following the LGB strategy, B_1 and B_2 bid for A_2 in the first $T = \left\lfloor \frac{p_{A_1}^{(0)} - p_{A_2}^{(0)}}{\epsilon} \right\rfloor$ steps. Let us assume that $p_{A_1}^{(T+1)} = p_{A_2}^{(T+1)}$ in the $(T+1)$st step, and B_2 placed the last bid on A_2. Then B_1 can randomly select one of the following two options.

B_1 might bid for A_1 and wins it for $p^{(0)}_{A_1}$, and B_2 wins A_2 for $p^{(0)}_{A_1} - \epsilon$, since both B_1 and B_2 have placed bids on auctions and there will be no one who could overbid them.

Additionally, B_1 perhaps bids for A_2 and overbids B_2. Hence, B_2 shall place a new bid. At this point, at $T + 2$, $p^{(T+2)}_{A_1} = p^{(0)}_{A_1}$ and $p^{(T+2)}_{A_2} = p^{(0)}_{A_1} + \epsilon$. In this case, B_2 will bid for A_1 as it is cheaper, and the auctions terminate: B_1 wins A_2 for $p^{(0)}_{A_1}$ and B_2 wins A_1 for $p^{(0)}_{A_1}$.

Let us assume that this statement is also true for $M - 1$ vehicles (and $N \geq M$ parking lots).

Now, we shall prove the statement for M vehicles. According to the induction assumption, after $T - \delta$ steps of the SIA, the first $M - 1$ vehicles are not overbid, and have bid for the cheapest $M - 1$ parking lots following the LGB strategy. Additionally, all of these $i \in \{1, 2, \dots M - 1\}$ auctions have a current $p^{(0)}_{(M-1)-} \leq p^{(T-\delta)}_i \leq p^{(0)}_{(M-1)-} + \epsilon$ price. However, the Mth vehicle is currently overbid in the $M - 1$ cheapest auctions in the $(T - \delta)$th step. Supposing that $p^{(0)}_{M-} \geq p^{(0)}_{(M-1)-} + \epsilon$, the Mth vehicle will bid for a parking space of which auction is inside the first $M - 1$ auctions as it is still cheaper than the Mth parking space. Therefore, in the $(T - \delta + 1)$st step, a vehicle that has not been overbid yet in the first $M - 1$ auctions, shall bid again on an auction making another vehicle bid in the $(T - \delta + 2)$th step. This chain reaction ensures that in $\delta = \left\lfloor \frac{p^{(0)}_{M-} - p^{(0)}_{(M-1)-}}{\epsilon} \right\rfloor$ additional steps the first $M - 1$ auctions will reach the $p^{(0)}_{M-}$ price. Consequently, in the Tth step, an overbid driver agent j will have two options:

Firstly, driver agent j might bid for parking lot with Mth cheapest starting price $p^{(T)}_{M-} = p^{(0)}_{M-}$. As the $M - 1$ other vehicles will not be overbid this way, it will win this parking lot for $p^{(0)}_{M-}$, and the others will pay either $p^{(T)}_i = p^{(0)}_{M-}$ or $p^{(T)}_i = p^{(0)}_{M-} + \epsilon$ for their parking spaces. Otherwise, the Mth parking lot would not be the cheapest for its $p^{(0)}_{M-}$ price.

Secondly, if j bids on any of the first $(M - 1)$ auctions for $p^{(T)}_{M-} = p^{(0)}_{M-}$ price, it will overbid another vehicle. It starts another chain reaction; however, at this time, the Mth auction will be a relevant alternative for the M vehicles; hence, all the vehicles will win a parking lot for either a $p^{(0)}_{M-}$ or a $p^{(0)}_{M-} + \epsilon$ price. □

APPENDIX B
PROOF OF THEOREM 3.

Proof. We prove the statement by induction.

Without loss of generality, consider a simple scenario with two parking lots (A_1, A_2), three vehicles (B_1, B_2, B_3), and $p_{v_{B_1}} > p_{v_{B_2}} > p_{v_{B_3}}$. Following the LGB strategy, B_1, B_2, and B_3 bid for A_1 and A_2. According to Lemma 1, after T steps, we reach a $p^{(T+1)}_{A_1} = p^{(T+1)}_{A_2} = p_{v,B_3} + \epsilon$ state. In this state, B_3 is no longer capable of making additional bids for any of the auctions. Furthermore, if B_1 and B_2 sent the last bids for A_1 and B_2, then they win A_1 and A_2 for a price of p_{v,B_3}. On the other hand, if B_3 *has* sent the last bid for for example A_1 for p_{v,B_3}, then, for example, B_1 is currently overbid. The B_1 vehicle has two options in the $(T+1)$th step.

It might bid for A_1, it will win it for the current $p^{(T+1)}_{A_1} = p_{v,B_3} + \epsilon$ price. As B_2 is currently winning on A_2 (for p_{v,B_3}), and B_3 cannot place further bids, the SIA terminates. On the other hand, if B_1 bids for A_2, it will overbid B_2 for $p^{(T+1)}_{A_2} = p_{v,B_3} + \epsilon$. It makes B_2 bid for A_1 for $p^{(T+2)}_{A_1} = p_{v,B_3} + \epsilon$. As B_3 cannot make any further bids, the SIA will terminate.

Now, let us assume that the statement is also true for $M - N - 2$ vehicles (and $N < M - 2$ parking lots).

To prove the statement for $M - N - 1$ vehicles and $N < M - 1$ parking lots, it is easy to see that after some $T - \delta$ steps, $M - N - 2$ vehicles with the $(M - N - 2)$nd lowest (or the $(N + 2)$nd highest) valuations will not be able to bid further, as $\forall i : p^{(T-\delta)}_i > p_{v,\left(M-N-2\right)^-}$. After additional δ steps, the current prices will reach the valuation of the $(M - N - 1)$st lowest (or the $(N + 1)$st highest) vehicle: $\forall i : p^{(T)}_i = p_{v,(M-N-1)^-} + \epsilon (= p_{v,(N+1)^+} + \epsilon)$. Regarding the vehicle with the $(N + 1)$st highest valuation is overbid or not, there are two possibilities.

If it is overbid and it cannot place any further bids, then the first N vehicles with the highest N valuations will win the SIA for the price of $p_{v,(N+1)^+}$.

If the vehicle is currently not overbid, then, in the $(T + 1)$st step, a vehicle with higher valuation shall pose another bid, causing a chain reaction, after which each of the first N vehicles with the highest valuations will win the auctions for $p_{v,(N+1)^+} + \epsilon$. □

APPENDIX C
PROOF OF THEOREM 4.

Proof. We will prove the statement by Petri nets [34]. A general Petri net could cover all transitions and places of a restricted, i.e., deterministically timed Petri nets that has identical structure and entry points to the general Petri net. Hence, it would be easy to see that the fully parallel SIA is a generalization of the serialized implementation if we can define two Petri nets having similar structures and initial positions: one general Petri net modeling the former, and a deterministic timed Petri net modeling the latter algorithm. In the following, we demonstrate how such Petri nets could be constructed.

To construct corresponding Petri nets, we shall track the bids and the possibility of bidding of the bidder agents. Let us model the possibility of bidding, i.e., the `Overbid` state of a bidder in Fig. 3, as B_j places in the Petri net. Moreover, $A_{i,j}$ will be places in the Petri net to represent that bidder j is currently in `Winning` state on auction i. Naturally, there shall be no bids on the auctions at the beginning; hence, *starting transitions* $t_{s,j}$ put exactly one token to each B_j places. In addition to the *starting* $t_{s,j}$ transitions, the rest of the transitions are defined between the two sets of places mentioned above. There are exactly two transitions between any B_j and $A_{i,j}$ pair of places. *Bidding transitions* $t_{bi,j}$ originated from the B_j places and pointing to $A_{i,j}$ places represent that the bidder j bids at the i auction. *Gating transitions* $t_{gi,j}$ originate from $A_{i,j}$ places and terminate in B_j places, represent the `tell_overbid` transitions in Fig. 3. To ensure that a bidder always wins at most one auction, and to also ensure that auctions run the LGB strategy for a single item demand, each bidding transition $t_{bi,j}$ passes tokens to

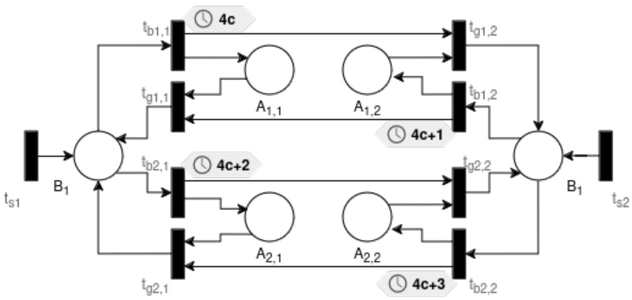

Fig. 8. Structure of the Petri net models. With deterministic timing, it represents the serialized version. Without the timing, the Petri net corresponds to the fully parallel SIA execution with LGB strategy for 1-item demand.

each $t_{gi,k}$ gate transition, where $k \in \{1, 2, \dots M\}, k \neq j$. The $t_{gi,k}$ gate transitions require at least two tokens on its inputs to release a single one. In this Petri net, if there is a token at the $A_{i,j}$ place, it means that the bidder j is currently winning on auction i. If there is a token at some B_j place, it means that the bidder j had been overbid, and it has not placed a newer bid since then. Fig. 8 demonstrates this Petri net structure for a simple $N = 2, M = 2$ case. This Petri net represents the SIA with an LGB strategy of [25].

Now, we shall focus on the necessary modifications of the untimed Petri net to represent the process defined in Algorithm 2. To model the iterations of line 6 and line 7 of Algorithm 2, we shall use a deterministic timed transitions Petri net. We assume that the transitions fire in an instant; however, they get enabled and disabled due to a periodic clock signal. Fundamentally, the outer iteration of line 6 prescribes that all transitions related to auction i shall precede the transitions of auction $i + 1$. Moreover, the inner iteration of line 7 requires that bidding transitions of bidder j shall precede the bidding transitions of bidder $j + 1$. Yielding bidding transitions $t_{bi,j}$ shall get enabled at exactly the $NMc+iM+j, c \in \{0, 1, \dots\}$ moments. As bidding transitions are required for firing the gating transitions, gating transitions can remain untimed ones. Fig. 8 shows an example of a simple $N = 2, M = 2$ case of the resulting timed Petri net.

As both Petri nets have a common structure and starting transitions, the timed Petri net is a restriction of the untimed version. Consequently, the untimed Petri net, corresponding to the method described in [25], can cover all transitions of the timed version. That yields that the parallelly running SIA with LGB strategy is a generalization of our serialized auction simulation method. $\square$

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Levente Alekszejenkó was born in Fehérgyarmat, Hungary, in 1996. He received an M.Sc. degree in computer science engineering from the Budapest University of Technology and Economics, Budapest, Hungary, in 2021.

Since 2021, he has been a Ph.D. student at the Department of Artificial Intelligence and Systems Engineering, Budapest University of Technology and Economics, Budapest, Hungary. His research focuses on multi-agent intelligent systems in the transportation network.

Tadeusz Dobrowiecki was born in Warsaw, Poland, in 1952. He received the M.Sc. degree in electrical engineering from the Technical University of Budapest, Budapest, Hungary, in 1975, the Ph.D. (candidate of sciences) degree in 1981, and the DSc (Doctor of Sciences) degree in 2018, both from the Hungarian Academy of Sciences.

He joined the Department of Artificial Intelligence and Systems Engineering, Budapest University of Technology and Economics, Budapest, Hungary. Currently he is a Professor Emeritus, teaching artificial intelligence and system identification.