An Ordered QR Decomposition based Signal Detection Technique for Uplink Massive MIMO System

Jyoti P. Patra, Bibhuti Bhusan Pradhan, and M. Rajendra Prasad

***Abstract*—Signal detection turns out to be a critical challenge in massive MIMO (m-MIMO) system due to the deployment of large number of antennas at the base station. Although, the minimum mean square error (MMSE) is one of the popular signal detection method, but, it requires matrix inversion with cubic complexity. In order to reduce computational complexity, several suboptimal signal detection methods were proposed such as Gauss-Seidel, successive over relaxation, Jacobi, Richardson methods. Although, these methods provide low complexity but their performance are limited to MMSE method. In this paper, we have proposed two signal detection techniques namely QR decompositions (QRD) and ordered QRD (OQRD). Finally, the performances of proposed signal detection methods are compared with various conventional methods in terms of symbol error rate (SER) and computational complexity. The simulation results validate that the proposed methods outperform the MMSE method with substantially lower computational complexity.**

***Index Terms*—Massive MIMO, Signal detction, QRD, OQRD, MMSE, Low complexity.**

I. INTRODUCTION

Massive multiple-input multiple-output (m-MIMO) is the most promising technique in 5G and beyond 5G (B5G) due to its high spectrum and energy efficiency, high spatial resolution, and simple transceiver design. In m-MIMO, a large number of antennas are employed at the base station (BS) [1, 2]. In the uplink transmission, the signals transmitted from mobile terminals are superimposed at the BS which cause interference and reduces the data rate. Due to deployments of large number of antennas, it requires advanced signal processing for data detection. The maximum-likelihood (ML) detection provides optimum bit error rate performance [1, 2]. However, it is not practically possible to employ the maximum likelihood (ML) detector due to its huge computational complexity as it searches all possible combination while performing data detection . The problem is also becoming more complicated when high-order modulation schemes are used and more users are multiplexed. Therefore, many nonlinear signal data detection methods are proposed which includes sphere decoder (SD) [3], tabu search (TS) [4], dirty paper coding [5] etc. Unfortunately, for massive MIMO systems with large number of antennas and higher-order modulation schemes, such methods need still very huge computation complexity. For spatially correlated massive MIMO system, random matrix theory based algorithms such as principal component analysis, eigen analysis, Karhunen-Loeve decomposition, were applied for signal detection [6–8]. However, Most researchers focused on linear signal detection algorithm rather non-linear methods for spatially uncorrelated m-MIMO system. Although, zero forcing and minimum mean square error (MMSE) are the two popular linear signal detection methods, they require matrix inversion with cubic complexity. Even though linear signal detection methods may not offer sufficient performance, still most of the researchers focused on linear methods because of reduced computational complexity. Several linear signal detection methods were proposed by exploiting the Gramian matrix to avoid matrix inversion which include Gauss-Seidel (GS) [9], the Neumann series (NS) [10], the successive overrelaxation (SOR) [11, 12], the Jacobi method [13], the conjugate gradient (CG) [14], the optimized coordinate descent (OCD), and the Richardson (RI) [15]. It has been observed that NS method is lower than the complexity of the detector based on GS, JA, RI and SOR methods, however its performance is the least. A hybrid pseudo-stationary iterative detection algorithm based on Chebyshev polynomial and Weyls inequality was proposed in [16] for uplink massive MIMO systems. This method provides near to ZF method. The authors in [17] proposed a weighted two stage (WTS) method which achieves similar performance to ZF method with lower complexity. Latter, a modified weighted two stage (MWTS) method was proposed in [18] which outperforms the WTS method. However, its performance is lower as compared to MMSE method. In [19], Cayley-Hamilton theorem-based two low complexity signal detection have been proposed to avoid the matrix inversion. This method has lower computational complexity as it does not involve in Gramian matrix. The authors in [20] performed signal detection by QR decomposition of Gram matrix $G = H^H H$. This method has performance limitation to ZF method because the QR decomposition was applied to ZF Gramian matrix. Similarly, in [21], the author applied several matrix decomposition technique such as QR, LDL and Cholesky. These matrix decomposition algorithm were applied to MMSE Grammian matrix, therefore their performances are limited to MMSE method.

In this paper, we proposed two signal detection methods namely QR decomposition (QRD) and order QRD (OQRD) methods for m-MIMO uplink communication system. The QR decomposition is directly applied to original channel matrix H

Jyoti P. Patra and M. Rajendra Prasad are with the Department of Electronics and Communication Engineering, Vidya Jyothi Institute of Technology, Hyderabad, Telangana, India (e-mail: jyotiprasannapatra@gmail.com, rajendraresearch@gmail.com).

Bibhuti Bhusan Pradhan is with the Department of Electronics and Communication Malla Reddy Engineering College, Hyderabad, Telangana, India (e-mail: bibhu.iisc@gmail.com).

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to obtain estimated transmitted signal. Thus, it outperforms other QR decomposition methods [20, 21] where the ZF and MMSE Gramain matrix are decomposed by QR method. Furthermore, the performance of QRD method increases by ordering the column norm of channel matrix in ascending manner. We call this method as OQRD method. The proposed QRD and OQRD methods are compared with various conventional method such as Gauss-Seidel, successive over relaxation, Jacobi, Richardson and MMSE methods in terms of symbol error rate and computational complexity.

We organize our paper as follows. In Section II, we describe the massive MIMO uplink system model. In Section III, we discuss various signal detection methods. In Section IV, we present the proposed signal detection methods. In Section V, we show simulation results of proposed and conventional methods in terms of symbol error rate. Finally, Section VI concludes the paper.

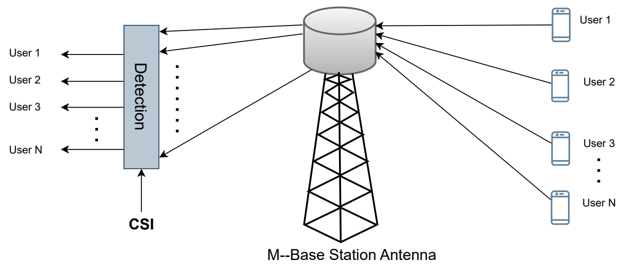

Fig. 1: Block diagram of uplink massive MIMO system with M number of base station antenna and N number of users

II. SYSTEM MODEL

The uplink channel is used to transmit data symbols from the user terminal to the base station. In a multiuser uplink massive MIMO system, M number of base station antennas are employed to serve N number of users simultaneously as shown in the Fig 1. Let $\boldsymbol{x}$ denote the complex valued $N \times 1$ simultaneously transmitted signal vector from the N users to the base station. The received signal vector $\boldsymbol{y}$ at the BS can be given by

$$\boldsymbol{y} = \mathbf{H}\boldsymbol{x} + \boldsymbol{n} \tag{1}$$

where $\mathbf{H}$ is the channel matrix between the user terminal and the base station with size $M \times N$ and $M > N$. The parameter $\boldsymbol{n}$ is the $M \times 1$ additive white Gaussian noise (AWGN). Although, the maximum likelihood (ML) method is the optimal signal detection method, it is not preferable from the hardware implementation perspective due to its high computational complexity. Therefore, suboptimal linear signal detection techniques such as zero forcing and minimum mean square error (MMSE) methods are widely accepted due to their near-optimal performance with lower computational complexity as compared to ML method. The signal detection based on MMSE method is given by

$$\hat{\boldsymbol{x}} = \left(\boldsymbol{H}^H\mathbf{H} + \frac{N}{SNR}\mathbf{I}_N\right)^{-1}\boldsymbol{H}^H\boldsymbol{y} = \mathbf{A}^{-1}\hat{\boldsymbol{x}}_{MF} \tag{2}$$

where $A = \left(\boldsymbol{H}^H\mathbf{H} + \frac{N}{SNR}\mathbf{I}_N\right)^{-1}$ and $\hat{\boldsymbol{x}}_{MF} = \boldsymbol{H}^H\boldsymbol{y}$. The matrix $\mathbf{I}_N$ is the $N \times N$ identity matrix and SNR is the signal to noise ratio. The MMSE method involves large matrix inversion operations with cubic complexity. To achieve close performance of MMSE with reduce complexity, several signal detection methods have been proposed such as Jacobi, Richardson, Gauss-Seidel, successive over relaxation methods by exploiting the Gram matrix.

III. CONVENTIONAL SIGNAL DETECTION METHOD

In this section, we have discussed various signal detection methods namely Jacobi, Richardson, Gauss-Seidel, successive over relaxation methods for massive MIMO uplink system.

A. Jacobi Method

The Jacobi method was proposed for m-MIMO uplink system in [13]. The Jacobi method approximate the matrix inversion with reduces complexity. The Jacobi method is an iterative approach for finding the solution to a diagonally dominant system. The equation (2) can be rewritten as

$$\mathbf{A}\hat{\boldsymbol{x}} = \hat{\boldsymbol{x}}_{MF} \tag{3}$$

Note that when N/M is large, matrix A becomes diagonally dominant. The estimated signal can be obtained as

$$\hat{\boldsymbol{x}}^{(\boldsymbol{n})} = \mathbf{D}^{-1}\left[\hat{\boldsymbol{x}}_{MF} + (\mathbf{D} - \mathbf{A})\hat{\boldsymbol{x}}^{(n-1)}\right] \tag{4}$$

where $\mathbf{D}$ is the digonal matrix of $\mathbf{A}$. The initial values can be selected as

$$\hat{\boldsymbol{x}}^{(0)} = \mathbf{D}^{-1}\hat{\boldsymbol{x}}_{MF}. \tag{5}$$

It can be verified that the first iteration of JA method does not involve matrix multiplication, thus computational complexity decreases.

B. Richardson Method

The Richardson method was proposed in [15]. In this method, the signal detection is performed by iterative process through the exploitation of Gramian matrix $\mathbf{G} = \mathbf{H}^H\mathbf{H}$. Here the convergence rate is very sensitive to a selection of relaxation parameter (ω) where $0 < \omega \leq \frac{2}{\lambda_{max}}$ and the optimum value of ω is defined as $w = \frac{2}{\lambda_{\min} + \lambda_{\max}}$. The parameter λ_{max} and λ_{min} are the maximum and minimum eigenvalues of the symmetric positive definite matrix $\mathbf{A}$ respectively.The estimated signal is obtained as

$$\boldsymbol{x}^{(n+1)} = \boldsymbol{x}^{(n)} + \omega\left[\boldsymbol{y} - \boldsymbol{H}\boldsymbol{x}^{(n)}\right] \quad n = 0, 1, 2, \cdots \tag{6}$$

If a prior knowledge of $\mathbf{x}^{(0)}$ is missing, a zero vector can be considered without loss of generality. It can also be selected as $\hat{x}^{(0)} = \mathbf{D}^{-1}\hat{\boldsymbol{x}}_{MF}$ and iteratively refined. The accuracy and the number of computations are highly affected by the value of ω.

C. Gauss-Sidel Method

The Gauss Sidel method computes the solution by an iterative behaviour where the Hermitian positive semi-definite matrix (**A**) is decomposed to a lower triangular matrix (**L**), upper triangular elements (**U**), and the diagonal entries (**D**). In other words, the matrix **A** can be written as

$$\mathbf{A} = \mathbf{D} + \mathbf{L} + \mathbf{U}. \tag{7}$$

This method performs the signal detection in an iterative process as given by

$$\hat{\boldsymbol{x}}^{(n)} = [\mathbf{D} + \mathbf{L}]^{-1}\left[\hat{\boldsymbol{x}}_{MF} - \mathbf{U}\hat{\boldsymbol{x}}^{(n-1)}\right], \quad n = 1, 2, \cdots I_T \tag{8}$$

where I_T is the total number of iterations. Typically, the initial data signal $\hat{\boldsymbol{x}}^{(0)}$ is considered as a zero vector for simplification.

D. Successive Over Relaxation Method

In order to avoid the large dimension inversion matrix, the successive over relaxation(SOR) is a best choice in signal detection. It improves the accuracy of GS method by using a relaxation parameter (ω). The signal is estimated as

$$\hat{\boldsymbol{x}}^{(n)} = \left[\frac{1}{\omega}\mathbf{D} + \mathbf{L}\right]^{-1}\left[\hat{\boldsymbol{x}}_{MF} + \left[\left[\frac{1}{\omega} - 1\right]\mathbf{D} - \mathbf{U}\right]\hat{\boldsymbol{x}}^{[n-1]}\right] \tag{9}$$

Convergence of the SOR method is highly affected by the relaxation parameter (ω). In the MIMO framework, the relaxation parameter (ω) of the SOR technique is typically good when $0 < \omega < 2$. The optimum value is given by $w = \frac{2}{1+\sqrt{1-a^2}}$ where $a = \left(1 + \sqrt{\frac{N}{M}}\right)^2 - 1$. This value of w is fixed throughout the iteration process.

IV. Proposed Signal Detection Methods

In this section, the proposed signal detection methods namely QRD and OQRD are discussed for uplink massive MIMO system.

A. QRD Method

In this paper, we have applied QR decomposition to the channel matrix **H** for performing signal detection. The relation between received and transmitted signal can be written in matrix form as given by

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_M \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & \cdots & h_{1N} \\ h_{21} & h_{22} & \cdots & h_{2N} \\ \vdots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \vdots \\ h_{M1} & h_{M2} & \cdots & h_{MN} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{bmatrix} + \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_M \end{bmatrix} \tag{10}$$

where h_{ij} is channel impulse response between jth transmitting antenna to the ith receiving antenna and $j = 1, 2, ..., N-1$ and $i = 1, 2, ..., M-1$. The channel matrix **H** can be decomposed into QR factors as $\mathbf{H} = \mathbf{QR}$ where $Q_{M\times N}$ is an orthonormal matrix and $R_{N\times N}$ is an upper triangular matrix. Substituting $\mathbf{H} = \mathbf{QR}$, the received signal can be written as

$$\mathbf{y} = \mathbf{Hx} + \mathbf{n} = \mathbf{QR}\boldsymbol{x} + \boldsymbol{n} \tag{11}$$

After multiplying $\mathbf{Q}^H$ with the received signal vector $\boldsymbol{y}$, equation (11) can be modified to

$$\tilde{y} = \mathbf{Q}^H\boldsymbol{y} = \mathbf{Q}^H(\mathbf{H}\boldsymbol{x} + \boldsymbol{n}) = \mathbf{R}x + \tilde{n} \tag{12}$$

The equation (12) can be expressed in matrix form as

$$\begin{bmatrix} \tilde{y}_1 \\ \tilde{y}_2 \\ \vdots \\ \tilde{y}_N \end{bmatrix} = \begin{bmatrix} R_{11} & R_{12} & \cdots & R_{1N} \\ 0 & R_{22} & \cdots & R_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & R_{NN} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{bmatrix} + \begin{bmatrix} \tilde{n}_1 \\ \tilde{n}_2 \\ \vdots \\ \tilde{n}_N \end{bmatrix} \tag{13}$$

Since, **R** is a lower triangular matrix, backward substitution method can be applied to obtain the estimated transmitted signal vector and can be written as

$$\hat{x}_N = \Pi[\tilde{y}_N / R_{NN}] \tag{14}$$

$$\hat{x}_k = \Pi\left[\frac{\tilde{y}_k - \sum\limits_{j=k+1}^{N}\tilde{y}_{kj}x_j}{R_{kk}}\right], k = N-1 : -1 : 1 \tag{15}$$

where $\Pi()$ denotes the hard decision function. The detail steps of QRD method is summarized below.

[*Step* 1]: Initialization: $\boldsymbol{y}$, $\mathbf{H}$, $\boldsymbol{x}$; $\boldsymbol{y} = \mathbf{H}\boldsymbol{x} + \boldsymbol{n}$

[*Step* 2]: Decomposition of channel matrix $\mathbf{H} = \mathbf{QR}$,
$\mathbf{y} = \mathbf{Hx} + \mathbf{n} = \mathbf{QR}\boldsymbol{x} + \boldsymbol{n}$

[*Step* 3]: Multiplication of $\mathbf{Q}^H$ with $\boldsymbol{y}$
$\tilde{\boldsymbol{y}} = \mathbf{Q}^H\boldsymbol{y} = \mathbf{R}x + \tilde{n}$

[*Step* 4]: Obtaining the transmitted signal using backward substitution method

$$\hat{x}_N = \Pi[\tilde{y}_N / R_{NN}]$$

$$\hat{x}_k = \Pi\left[\frac{\tilde{y}_k - \sum\limits_{j=k+1}^{N}\tilde{y}_{kj}x_j}{R_{kk}}\right], k = N-1 : -1 : 1$$

B. OQRD Method

The QRD method may suffer from error propagation problem if the initial signal is not detected correctly. Therefore, an order QRD (OQRD) method is proposed which orders the column vector of the channel matrix **H**. The relationship between received and transmitted signal can be written in the column form as

$$\boldsymbol{y} = \mathbf{H}\boldsymbol{x} + \boldsymbol{w} = \boldsymbol{h}_1x_1 + \boldsymbol{h}_2x_2 + \cdots + \boldsymbol{h}_Nx_N + \boldsymbol{n} \tag{16}$$

where $\boldsymbol{h}_k$ is the kth column vector of channel matrix **H** and x_k is the kth element of the transmitted signal vector $\boldsymbol{x}$. To perform the ordering of column vector in an ascending manner, the procedure is as given follows. At first, we calculate the norm of each column vector of the $\mathbf{H}$ matrix as given by

$$norm_k = \|h_k\| \qquad k = 1, 2, ..., N \tag{17}$$

Then, we sort vector $\boldsymbol{norm} = [\ norm_1,\ \ norm_2,\ \ \cdots\ \ , norm_N\ ]$ in an ascending manner and find the $\boldsymbol{index}$ term

$$\boldsymbol{index} = sort(\boldsymbol{norm}) \tag{18}$$

The column vectors are arranged according to the indices to obtain the ordered banded CFR matrix $\mathbf{H}_o$

$$\boldsymbol{h}_{o,k} = \mathbf{H}_o(:, k) = \mathbf{H}(:, index_k) \qquad k = 1, 2, ..., N \tag{19}$$

where $\boldsymbol{h}_{o,k}$ is the kth column vector of $\mathbf{H}_o$ matrix and $index_k$ denotes the kth element of $\boldsymbol{index}$ vector. The ordering of the transmitted signal vector can also be represented in terms of the indices as given by

$$x_{o,k} = x(index_k) \qquad k = 1, 2...N \tag{20}$$

Substituting the ordered channel matrix $\mathbf{H}_o$ as defined in (19) and ordered transmitted signal vector $\boldsymbol{x}_o$ (20), the received signal vector $\boldsymbol{y}$ can be expressed as

$$\boldsymbol{y} = \mathbf{H}_o\boldsymbol{x}_o + \ \boldsymbol{n} \tag{21}$$

The order channel matrix $\mathbf{H}_o$ can be decomposed into QR factorization $H_o = Q_o R_0$. After multiplication $\mathbf{Q}_o^H$ on both sides of (21), it yields

$$\begin{aligned} \tilde{\mathbf{y}} &= \mathbf{Q}_o^H \boldsymbol{y} = \mathbf{Q}_o^H(\mathbf{H}_o\boldsymbol{x}_o + \boldsymbol{n}) \\ &= \mathbf{Q}_o^H(\mathbf{Q}_o\mathbf{R}_o\boldsymbol{x}_o + \boldsymbol{n}) = \mathbf{R}_o\boldsymbol{x}_o + \tilde{\mathbf{n}} \end{aligned} \tag{22}$$

The transmitted signal is obtained after performing the backward substitution method. The detail steps of QRD method is summarized below.

[*Step* 1]: Initialization: $\boldsymbol{y}$, $\mathbf{H}$, $\boldsymbol{x}$

$$\boldsymbol{y} = \mathbf{H}\mathbf{x} + \boldsymbol{w} \ \ = \boldsymbol{h}_1 x_1 + \boldsymbol{h}_2 x_2 + \ \cdots \ + \boldsymbol{h}_N x_N + \boldsymbol{n}$$

[*Step* 2]: Calculating norm of Channel matrix $\mathbf{H}$ and index the norm in ascending order

$$norm_k = \|h_k\| \ , \qquad k = 1, 2, ..., N$$
$$\boldsymbol{index} = sort(\boldsymbol{norm})$$

[*Step* 3]: Modifying the channel matrix $\mathbf{H}$ and signal vector $\boldsymbol{x}$ in terms of ascending order

$$\mathbf{H}_o(:, k) = \mathbf{H}(:, index_k) \quad k = 1, 2, ..., N$$
$$x_{o,k} = x(index_k) \qquad k = 1, 2..., N$$

[*Step* 4]: Decomposition of channel matrix $\mathbf{H}_o = \mathbf{Q}_o\mathbf{R}_0$,

$$\mathbf{y} = \mathbf{H}_o\mathbf{x}_o + \mathbf{n} = \mathbf{Q}_o\mathbf{R}_o\boldsymbol{x}_o + \boldsymbol{n}$$

[*Step* 5]: Multiplication of $\mathbf{Q}_o^H$ with $\boldsymbol{y}$

$$\tilde{\mathbf{y}} = \mathbf{Q}_o^H\boldsymbol{y} = \mathbf{R}_o\boldsymbol{x}_o + \tilde{\mathbf{n}}$$

[*Step* 6]: Obtaining the transmitted signal using backward substitution method

$$\hat{x}_{oN} = \Pi[\tilde{y}_{oN}/R_{oNN}]$$

$$\hat{x}_{ok} = \Pi\left[\frac{\tilde{y}_{ok} - \sum\limits_{j=k+1}^{N} \tilde{y}_{okj} x_{oj}}{R_{okk}}\right], k = N-1 : -1 : 1$$

This method eliminates the error propagation problem of QRD by detecting the stronger signal at first and then cancels its effects before detection of weaker signal.

C. Computational Complexity

In this section, the computational complexity of the proposed QRD, OQRD methods are analysed in terms of multiplications. Then, the complexity of the proposed metod is compared with various signal detection methods which includes MMSE, Jacobi, Richardson, Gauss Seidel and SOR methods. The QR decomposition of channel matrix H requires $N^{2.529}$ [22]. Multiplying Q^H with Y requires NM^2 complexity. To obtain the estimated transmitted data signal requires backward substitution algorithm as given in [step 4] of the proposed QR method requires $2N(N-1)$ complexity. Thus, the QRD method requires a total of $N^{2.529} + 4NM^2 + 2N(N-1)$ complexity. The ordered QRD (OQRD) require same complexity as QRD method. In addition to that, OQRD method requires to find the norm of the column vector of matrix $\mathbf{H}$ which needs $4MN$ operations. Thus, total complexity involves in OQRD method is $N^{2.529} + 4NM^2 + 2N(N-1) + 4MN$. The computational complexity of the proposed methods are compared with conventional methods and is given in Table 1.

TABLE I
COMPUTATIONAL COMPLEXITY

Method	Multiplications
MMSE	$2MN^2 + (10/3)N^3 + 4MN + 4N^2$
Jacobi [13]	$(4M + 4I_T + 1)N^2 + 2NM$
Richardson [15]	$(4M + 4I_T)N^2 + 2NM$
GS [9]	$(4M + 4I_T - 2)N^2 + 2(N - 2I_T + 1)N$
SOR [12]	$(4M + 4I_T - 2)N^2 + 2(M - I_T + 1)N$
QRD	$N^{2.529} + 4NM^2 + 2N(N-1)$
OQRD	$N^{2.529} + 4NM^2 + 2N(N-1) + 4MN$

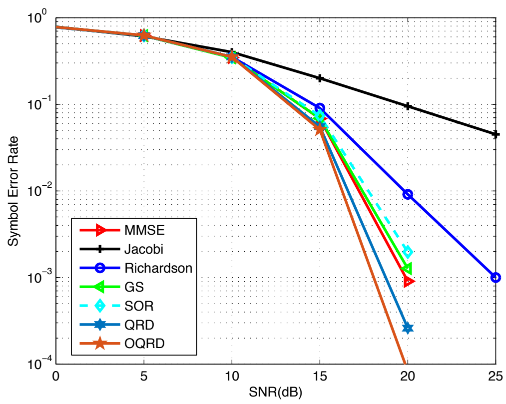

Fig. 2: SER performance comparison of various signal detection method forr NR = 24, NT = 64

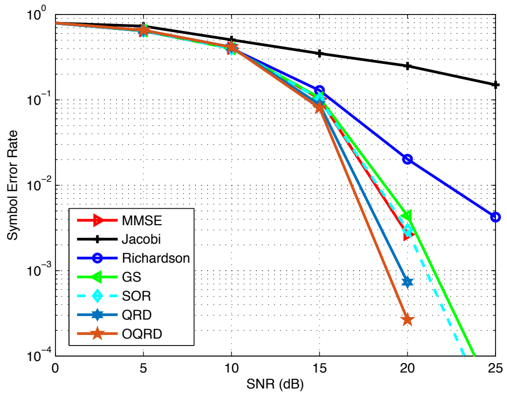

Fig. 3: SER performance comparison of various signal detection method for NR = 36, NT = 64

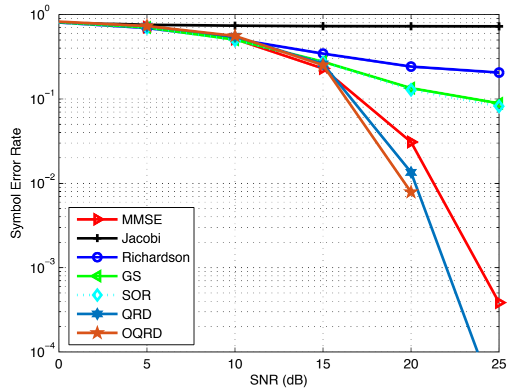

Fig. 4: SER performance comparison of various signal detection method for NR = 48, NT = 64

V. RESULTS

In this section, the performance of proposed QRD and OQRD methods are compared with various conventional signal detection methods for uplink massive MIMO system in terms of symbol error rate (SER) and computational complexity. The SER performance of various signal detection methods are carried out based on Monte Carlo simulation using MATLAB. We have considered $M = 64$ number of receiving antennas at the base station and N number of users with each user equipped with single transmitting antenna. For simulation, the antenna configuration $(N \times M)$ are as follows: 24×64, 36×64 and 48×64. The baseband signal modulation technique uses 16QAM, and for each SNR value, we simulate at least 48000 symbols. The transmission channel is considered as non-correlated Rayleigh fading channel. The perfect channel state information (CSI) is assumed to be known at the receiver terminal.

Fig 2, Fig.3 and Fig 4 show the SER performance comparison of proposed QRD and OQRD methods with various conventional signal detection methods for number of users $N = 24$, $N = 36$ and $N = 48$ respectively. From the simulation results it is seen that the Jacobi method has significantly lower performance. It is observed that the performance of Richardson method is much better Jacobi methods. The simulation results shows that the performance of SOR significantly improves and outperforms all the conventional methods when the number of users increases. The GS method is much better than Jacobi and Richardson methods. The performance of SOR provides slightly better when the ratio between user to BS i.e. N/M increases. Since, all the signal detection methods are derived from MMSE method based on several approximate matrix inversion methods, therefore, their performance are always lower than the MMSE methods. The performance of proposed QRD significantly outperforms the MMSE method. It is observed that the performance of OQRD method gives better performance than QRD method as it performs addition ordering of the channel matrix.

The SER vs number of users (N) performance comparison for various signal detection methods at $20dB$ SNR is shown in the Fig 5. The simulation results shows Jacobi and Richardson achieves considerable performance for lower number of users. But, as the number of users increases their performance significantly decreases. It is observed that the performance of GS and SOR methods performs close to MMSE method for lower number of users and the performance gap increase with a large number of users. It can be seen that the performance of proposed QRD method outperforms the MMSE method for lower to medium number of users. From the result, it is also observed that the performance of proposed OQRD method significantly outperform the MMSE method. Although the gap between OQRD and MMSE method decreases with very high number of users but still the OQRD method is significantly outperforms the MMSE method.

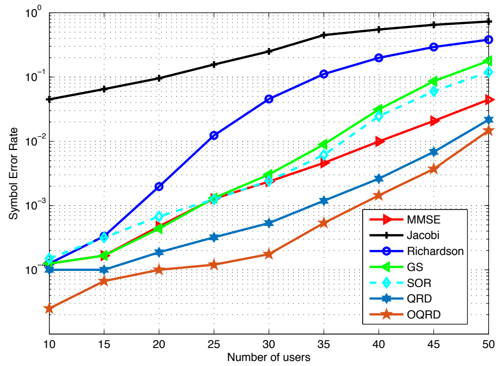

Fig. 5: SER Vs number of user (N)performance comparison of various signal detection method at 20dB SNR

VI. CONCLUSION

In this paper, we have proposed QRD and OQRD based signal detection methods for massive MIMO uplink system. The QRD method is based on the QR factorization of the original channel matrix to obtain estimated transmitted signal. Furthermore, the OQRD method is proposed which enhances the performance of QRD method. The OQRD method is based on the QR decomposition of the column norm ordering of the channel matrix. These proposed methods are compared with various conventional signal detection methods which include Jacobi, Richardson, Gauss-Sidel, SOR and MMSE in terms of SER and computational complexity. The simulation results show that the proposed methods significantly outperforms conventional signal detection method with complexity lower than MMSE method. Therefore, the proposed OQRD method can be considered as a suitable signal detection technique for uplink massive MIMO system.

REFERENCES

[1] M. A. Albreem, W. Salah, A. Kumar, M. H. Alsharif, A. H. Rambe, M. Jusoh, and A. N. Uwaechia, "Low complexity linear detectors for massive mimo: A comparative study," *IEEE Access*, vol. 9, pp. 45 740 –45 753, 2021, DOI: 10.1109/ACCESS.2021.3065923.

[2] M. A. Albreem, M. Juntti, and S. Shahabuddin, "Massive mimo detection techniques: A survey," *IEEE Communications Surveys & Tutorials*, vol. 21, no. 4, pp. 3109–3132, 2019, DOI: 10.1109/COMST.2019.2935810.

[3] T. Datta, N. Srinidhi, A. Chockalingam, and B. S. Rajan, "Random-restart reactive tabu search algorithm for detection in large-mimo systems," *IEEE Communications Letters*, vol. 14, no. 12, pp. 1107–1109, 2010, DOI: 10.1109/LCOMM.2010.101210.101587.

[4] L. G. Barbero and J. S. Thompson, "Fixing the complexity of the sphere decoder for mimo detection," *IEEE Transactions on Wireless communications*, vol. 7, no. 6, pp. 2131–2142, 2008, DOI: 10.1109/TWC.2008.060378.

[5] C. KurisummoottiL Thomas and D. Slock, "Reduced-order zero-forcing beamforming vs optimal beamforming and dirty paper coding and massive mimo analysis," in *2018 IEEE 10th Sensor Array and Multichannel Signal Processing Workshop (SAM)*. IEEE, 2018, pp. 351–355, DOI: 10.1109/SAM.2018.8448769.

[6] L. Sanguinetti, E. Björnson, and J. Hoydis, "Toward massive mimo 2.0: Understanding spatial correlation, interference suppression, and pilot contamination," *IEEE Transactions on Communications*, vol. 68, no. 1, pp. 232–257, 2020, DOI: 10.1109/TCOMM.2019.2945792.

[7] "Eigen analysis of flipped toeplitz covariance matrix for very low snr sinusoidal signals detection and estimation," *Digital Signal Processing*, vol. 129, p. 103 677, 2022, DOI: 10.1016/j.dsp.2022.103677.

[8] O. Sharifi-Tehrani, M. F. Sabahi, and M. Raees Danaee, "Efficient gnss jamming mitigation using the marcenko pastur law and karhunenloeve decomposition," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 58, no. 3, pp. 2291–2303, 2022, DOI: 10.1109/TAES.2021.3131400.

[9] L. Dai, X. Gao, X. Su, S. Han, I. Chih-Lin, and Z. Wang, "Low-complexity soft-output signal detection based on gauss–seidel method for uplink multiuser large-scale mimo systems," *IEEE Transactions on Vehicular Technology,* vol. 64, no. 10, pp. 4839–4845, 2014, DOI: 10.1109/TVT.2014.2370106.

[10] L. Shao and Y. Zu, "Joint newton iteration and neumann series method of convergence-accelerating matrix inversion approximation in linear precoding for massive mimo systems," *Mathematical Problems in Engineering*, vol. 2016, 2016, DOI: 10.1155/2016/1745808.

[11] C. N. Cuong, T. T. Hong, and L. D. Khai, "Hard-ware implementation of the efficient sor-based massive mimo detection for uplink," in *2019 IEEE-RIVF International Conference on Computing and Communication Technologies (RIVF)*. IEEE, 2019, pp. 1–6, DOI: 10.1109/RIVF.2019.8713667.

[12] X. Gao, L. Dai, Y. Hu, Z. Wang, and Z. Wang, "Matrix inversion-less signal detection using sor method for uplink large-scale mimo systems," in *2014 IEEE Global Communications Conference*. IEEE, 2014, pp. 3291–3295, DOI: 10.1109/GLOCOM.2014.7037314.

[13] J. Zhou, Y. Ye, and J. Hu, "Biased mmse soft-output detection based on jacobi method in massive mimo," in *2014 IEEE International Conference on Communiction Problem-solving*. IEEE, 2014, pp. 442–445, DOI: 10.1109/ICCPS.2014.7062317.

[14] Y. Hu, Z. Wang, X. Gaol, and J. Ning, "Low-complexity signal detection using cg method for uplink large-scale mimo systems," in *2014 IEEE international conference on communication systems*. IEEE, 2014, pp. 477–481, DOI: 10.1109/ICCS.2014.7024849.

[15] B. Kang, J.-H. Yoon, and J. Park, "Low-complexity massive mimo detectors based on richardson method," *ETRI Journal*, vol. 39, no. 3, pp. 326–335, 2017, DOI: 10.4218/etrij.17.0116.0732.

[16] S. Shafivulla and A. Patel, "Linear complexity zf-based linear precoder for massive-mimo systems," in *2021 IEEE 26th International Workshop on Computer Aided Modeling and Design of Communication Links and Networks (CAMAD)*. IEEE, 2021, pp. 1–6, DOI: 10.1109/CAMAD52502.2021.9617813.

[17] Y. Liu, J. Liu, Q. Wu, Y. Zhang, and M. Jin, "A near-optimal iterative linear precoding with low complexity for massive mimo systems," *IEEE Communications Letters*, vol. 23, no. 6, pp. 1105–1108, 2019, DOI: 10.1109/lcomm.2019.2911472.

[18] M. Chinnusami, C. Ravikumar, S. Priya, A. Arumainayagam, G. Pau, R. Anbazhagan, P. S. Varma, and K. Sathish, "Low complexity signal detection for massive mimo in b5g uplink system," *IEEE Access*, 2023, DOI: 10.1109/ACCESS.2023.3266476.

[19] N. Jain, V. A. Bohara, and A. Gupta, "Pci-mf: Partial canonical identity and matrix factorization framework for channel estimation in mmwave massive mimo systems," *IEEE Open Journal of Signal Processing*, vol. 1, pp. 135–145, 2020, DOI: 10.1109/OJSP.2020.3020002.

[20] A. Boukharouba, M. Dehemchi, and A. Bouhafer, "Low-complexity signal detection and precoding algorithms for multiuser massive mimo systems," *SN Applied Sciences*, vol. 3, no. 2, p. 169, 2021, DOI: 10.1007/s42452-020-04085-z.

[21] S. Shahabuddin, M. H. Islam, M. S. Shahabuddin, M. A. Albreem, and M. Juntti, "Matrix decomposition for massive mimo detection," in *2020 IEEE Nordic Circuits and Systems Conference (NorCAS)*. IEEE, 2020, pp. 1–6, DOI: 10.1109/NorCAS51424.2020.9264998.

[22] C. Camarero, "Simple, fast and practicable algorithms for cholesky, lu and qr decomposition using fast rectangular matrix multiplication," *arXiv preprint*, 2018, DOI: 10.48550/arXiv.1812.02056.

Jyoti Prasanna Patra received his Ph.D. and M.Tech. degrees from the National Institute of Technology Rourkela, India in 2018 and 2012, respectively. He has completed his B.Tech. degree from Biju Patnaik University of Technology, Odisha, India in 2008. Currently, he is working as an Associate Professor in the Department of Electronics and Communication Engineering at Vidya Jyothi Institute of Technology, Hyderabad, Telangana India. His research interests include signal processing for wireless communication.

Bibhuti Bhusan Pradhan received his Ph.D. and M.Tech. degrees from the National Institute of Technology Rourkela, India, in 2018 and 2012, respectively. He has completed his B.Tech. degree from BPUT University, Odisha, India, in 2007. He is currently working as a Senior Assistant Professor in the Department of Electronics and Communication Engineering at Malla Reddy Engineering College, Hyderabad, Telangana India. His research interests concentrate on performance analysis of wireless communication systems over fading channels.

M. Rajendra Prasad obtained his B. Tech from SK University, M.E from Osmania University and was awarded Ph.D. in Internet of Things from Osmania University. He is currently serving as Professor and Head of the Department of Electronics and Communication Engineering at Vidya Jyothi Institute of Technology, Hyderabad, Telangana India. His main research interests are Security algorithms for IoT Networks and Embedded System Development for mobile applications.